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P.V. LUKIANOV, Ye. V. UTIUHOV

National University «Kyiv Aviation Institute», Kyiv, Ukraine

ABSENSE OF PHYSICAL CORRESPONDENCE AND REVERSIBILITY BETWEEN PRESSURE GRADIENT AND GRADIENT FREE LAMINAR COMPRESSIBLE BOUNDARY LAYER FLOWS

This paper examines the physical validity of existing theories regarding the laminar boundary layer of a compressible fluid. It is shown that while the equations describing fluid motion in classical studies (by Blasius and Kármán) correspond to the problem of a body moving through a fluid that is still at infinity, the boundary conditions employed actually correspond to the inverse problem of a fluid flowing around an immobile body (such as flow in a wind tunnel). Thus, there is a physical discrepancy between the theories and actual flows (problems). It is shown that a complete reversibility of the problems is impossible, as the results of one flow cannot be used to describe another due to the differing nature (dynamics) and kinematics of these flows. The physical formulations of the direct problem (motion of a body in a fluid) and the inverse problem (fluid flow around an immobile body), along with their numerical solutions, are presented. For the inverse problem, the velocity field is compared with known results (von Kármán's), while for the direct problem, it is compared with other studies. Overall, the qualitative results for both problems are similar to those used for comparison.

Keywords: *boundary layer, laminar compressible flow, pressure gradient and gradient free flows, external flow, Navier-Stokes equations.*

Introduction. The theory of the boundary layer—formed as a result of fluid friction against a solid surface—is of critical importance for aircraft, helicopters, and rockets. While the theory of incompressible boundary-layer flow [1] can be used as an approximation for small aircraft or unmanned aerial vehicles (UAVs) moving at speeds up to 100 m/s (a Mach number of approximately 0.3), it becomes necessary to account for the compressibility of the fluid (air) [2] in the case of aircraft and other flying vehicles with surface speeds reaching several hundred meters per second. At higher speeds (supersonic and hypersonic flow), fluid thermal conductivity and heat and mass transfer effects can no longer be neglected.

Analysis of recent research and publications. Previous studies [1, 2, 3, 4] have shown that, for incompressible laminar flow in the boundary layer, molecular viscosity cannot be assumed constant; otherwise, the theory (the linear solution to the Navier-Stokes equations) fails to match experimental data (which exhibit an exponential dependence). A detailed explanation of this fact is provided in [1]. However, even authoritative sources (see Schlichting's monograph [5], end of Section 67) note that to date, no one—including Stokes himself [6]—has fully proven the constancy of molecular viscosity throughout the entire domain of incompressible flow.

Unlike the studies just mentioned, the present work focuses on compressible laminar flow (Mach number 0.5–5) within the boundary layer. In this case, molecular viscosity is inherently variable. Nevertheless, a thorough review of existing literature reveals that authors consistently assume the equivalence of two problems: the motion of a body through a fluid that is still at infinity (e.g., the flight of an airplane, helicopter, or rocket) and the flow of a fluid past an immobile body (e.g., wind tunnel experiments)

[5, 7, 8]. For instance, in his work [7], von Kármán does not consider a pressure gradient in the equation of motion (or momentum conservation)—which corresponds to the direct problem of a body in motion; instead, the figures show that the flow around an immobile half-plane is actually being considered. This is evidenced by Fig. 5 in the cited work, where the velocity at the plate surface is zero. Thus, there is a common error in interpreting the flow physics: the assumption that the equation of motion for a body in a fluid (where no pressure gradient exists) can be applied to the problem of fluid flow over an immobile plane. Blasius's theory [9] serves as confirmation of the widespread nature of this error. Furthermore, O'Donnell [10] presents results from laboratory experiments (Figs. 5–9) that clearly indicate a case of fluid flowing around an immobile body (where the velocity at the body's surface is zero). This inconsistency is encountered even in recent studies. For instance, when investigating the boundary layer in flow past a flat plate, the authors of [11] present an equation (see 2.1 in [11]) that omits the longitudinal pressure gradient, even though the boundary conditions (see 2.3–2.4 in [11]) clearly indicate that the plate is immobile. The authors most likely aimed to obtain results that would correlate in some way with the work of Howarth [12]. However, in modern studies—such as those by Nobutaka Taniguchi et al. [13]—the longitudinal pressure gradient is already included in the momentum conservation equation.

Among the theoretical studies specifically addressing the problem of body motion subject to boundary conditions—and the generation of a laminar compressible boundary layer by that motion—one can cite the paper by Ackroyd (see equations 2–4 [14]). However, that work focuses on a slightly different problem. Furthermore, it does not present explicit velocity plots. The paper by Takhar et al. [15] is also of interest. Its value and importance lie in the fact that, firstly, it considers the direct problem of body motion and, secondly, it presents the same two dimensionless quantities as the present study: longitudinal velocity and temperature. Thus, despite the use of self-similar variables in [15], it is possible to compare the results to some extent. However, one must pay attention to the parameter representing the ratio of the wall temperature to the temperature at infinity, as presented in the aforementioned study.

The fundamental discrepancy lies in the absence of a force that would set the fluid in motion. Recognizing that the boundary layer on a moving body is gradient-free ($dp/dx = 0$), researchers invariably assume the longitudinal gradient to be zero in their work. They then proceed on the implicit assumption that the problem is reversible—treating the flow past an immobile body instead of the body's motion through a fluid that is still at infinity. The fact is such a flow cannot physically occur, as there is no driving force $dp/dx < 0$. Furthermore, the energy balance equation for a gradient flow must include a convective term arising from the total time derivative of pressure. Thus, it should be noted that for a compressible, heat-conducting flow, the distinction between gradient and gradient-free flows manifests in two terms of differing physical nature. Most importantly, results obtained from the gradient-free model are compared with experimental measurements from wind tunnels, where the flow is generated specifically by a longitudinal pressure gradient [16, 17, 18].

Calculations are also frequently simplified in studies—such as [19]—to facilitate their subsequent implementation in production. Specifically, the study [20] assumed that longitudinal thermal conductivity is negligible and that viscosity is a function solely of temperature. Meanwhile, the study [21] does not present the temperature dependence of viscosity for laminar flow and contains an incorrect comparison of theoretical data with experimental results. Furthermore, theoretical studies in this area face scalability

issues, as seen in [22]. Works [23] and [24] serve as important foundations for the present study; these works correctly applied boundary conditions. However, studies [24] and [25] address a more complex problem—namely, unsteady flow.

Problem statement. To obtain both analytical and numerical solutions describing laminar compressible heat-conducting fluid flow in the boundary layer for two problems: the motion of a body (direct problem) and the flow of fluid past an immobile body (reverse problem).

The aim of the study is to demonstrate, through a comparison of solutions, that no reversibility exists between the aforementioned problems and—crucially—that it is impossible to consider a problem in which the equations correspond to the motion of a body while the boundary conditions correspond to the flow around an immobile body. Such a problem is simply unphysical, as it does not correspond to any possible fluid flow.

Boundary layer on an immobile plane. According to the general system of equations, for a one-dimensional flow (where all derivatives except the longitudinal pressure gradient are identically zero) described by a single velocity component, we have:

$$0 = -\frac{1}{k\mathrm{Ma}^2}\frac{dp'}{dx'} + \frac{d}{dy'}\left(\mu'\frac{du'}{dy'}\right), \tag{1}$$

$$0 = \frac{k-1}{k}u'\frac{dp'}{dx'} + (k-1)\mathrm{Ma}_\infty^2\mu'\left(\frac{du'}{dy'}\right)^2 + \frac{1}{\sigma}\frac{d}{dy'}\left(\mu'\frac{dT'}{dy'}\right) \tag{2}$$

$$\mu' = (T')^{3/2}\left(\frac{1+T^*}{T'+T^*}\right) \tag{3}$$

The system of equations (1)–(3) comprises the momentum conservation equation (1), the energy balance equation (2), and the temperature dependence of viscosity (3) (Sutherland's formula). A distinctive feature of this system is the presence of a longitudinal pressure gradient, which is not accounted for in boundary-layer theories [7, 9].

If the flow domain is infinite (which is, of course, an approximation), the longitudinal pressure gradient can be treated as a constant:

$$\frac{dp'}{dx'} = -1. \tag{4}$$

Equation (1), taking into account (4) and the boundary condition

$$\frac{du'}{dy'}\Big|_{y'=y_{BL}} = 0 \tag{5}$$

transforms into the following equation:

$$\frac{du'}{dy'} = \frac{1}{k\mathrm{Ma}_\infty^2(T')^{3/2}\left(\frac{1+T^*}{T'+T^*}\right)}\left(y_{BL}-y'\right) \tag{6}$$

Equation (2), taking (6) into account, takes the following form:

$$0 = \left(\frac{k-1}{k}\right)\left((1-y')\frac{du'}{dy'} - u'\right) + \frac{1}{\sigma}\frac{d}{dy'}\left(\mu'\frac{dT'}{dy'}\right) \tag{7}$$

If we now substitute the explicit form for molecular viscosity (3) into equations (6) and (7), we obtain the following system of equations:

$$\frac{du'}{dy'}=\frac{1}{k\mathrm{Ma}_\infty^2(T')^{3/2}\left(\frac{1+T^*}{T'+T^*}\right)}(1-y') \tag{8}$$

$$0=\left(\frac{k-1}{k}\right)\left((1-y')\frac{du'}{dy'}-u'\right)+\frac{1}{\sigma}\frac{d}{dy'}\left((T')^{3/2}\left(\frac{1+T^*}{T'+T^*}\right)\frac{dT'}{dy'}\right) \tag{9}$$

The system of equations (8)–(9) is closed, as it involves only two unknown dimensionless functions: a velocity component and the absolute temperature.

To solve the problem, the necessary number of physical boundary conditions must be added to equations (8)–(9).

The presence of a first derivative in equation (8) allows for the formulation of only one physical boundary condition for the velocity. Clearly, this is the no-slip (or zero-slip) condition:

$$u'\Big|_{y'=0}=0 \tag{10}$$

The second physical boundary condition for velocity has already been accounted for in relation (5). Its physical significance lies in the absence of shear stresses, resulting from the velocity field reaching an asymptotic steady-state value.

For the temperature field—where the value T_∞ is chosen as the scale—we obtain the dimensionless temperature value at the wall

$$T'\Big|_{y'=0}=T'_W \tag{11}$$

and the temperature value at the outer edge of the boundary layer

$$T'\Big|_{y'=y_{BL}}=1\cdot \tag{12}$$

The system of differential equations (8)–(9), subject to boundary conditions (10), (11), and (12), was solved numerically. The constants k and σ were set to 1.4 and 0.7, respectively; these values are characteristic of air.

The results of the numerical calculations are presented in Figs. 1 and 2.

Figures 1 and 2 show that the dimensionless boundary-layer thickness increases with increasing velocity. This result is fully consistent with the findings of von Kármán [7], where the boundary-layer thickness also increases significantly with increasing velocity (Mach number).

Boundary layer on a moving plane. From the first fundamental form of the dimensionless equations for a steady, laminar, plane boundary layer in a viscous gas at high speeds, we have:

$$0=\frac{d}{dy'}\left(\mu'\frac{du'}{dy'}\right),\ 0=\frac{d}{dy'}\mu'\frac{d}{dy'}\left(h'+\frac{k-1}{2}\mathrm{Ma}_\infty^2u'^2\right)+\left(\frac{1}{\sigma}-1\right)\frac{d}{dy'}\left(\mu'\frac{dh'}{dy'}\right).$$

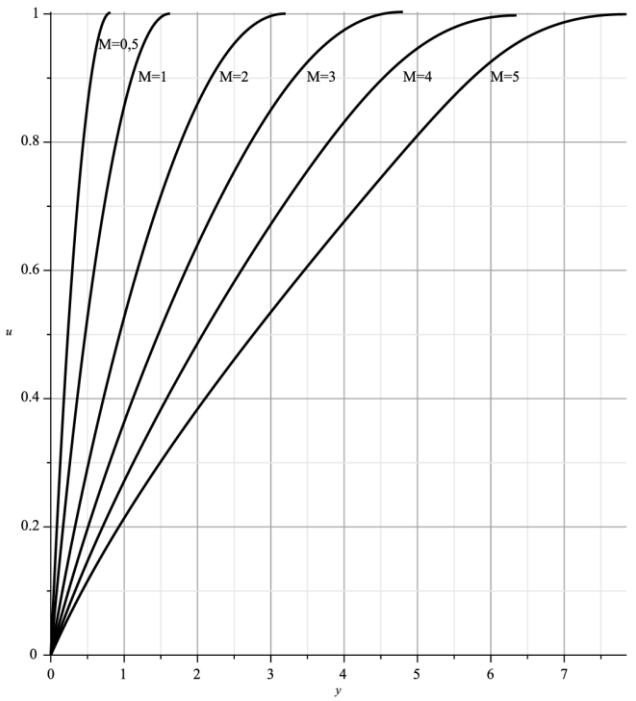


Fig. 1. Distributions of dimensionless velocity values for Mach numbers 0.5, 1, 2, 3, 4, and 5.

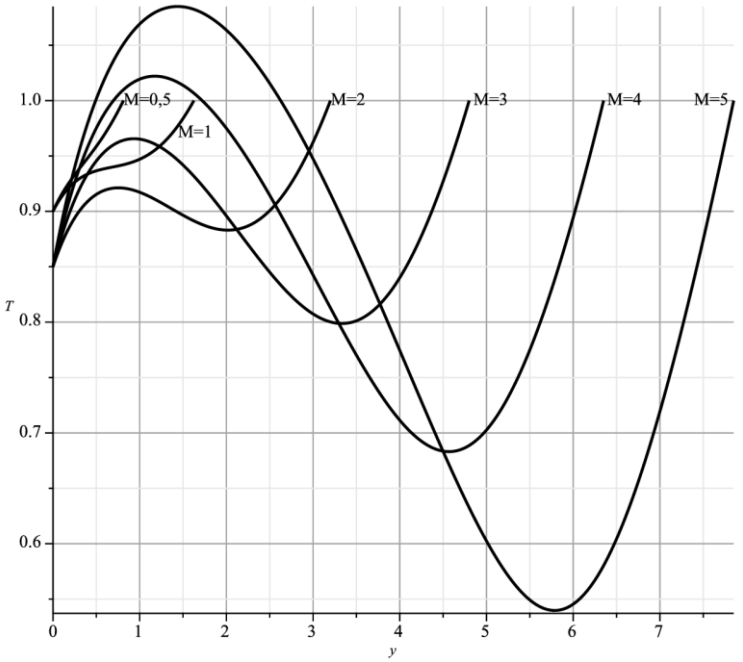


Fig. 2. Distributions of dimensionless temperature values for Mach numbers 0.5, 1, 2, 3, 4, and 5.

(13)

Viscosity μ' is still approximated by Sutherland's formula. From the first equation of system (13), we have:

$$\mu' \frac{du'}{dy'} = \text{Const} = \tau_0 \tag{14}$$

The second equation of system (13) can also be integrated once:

$$\mu' \frac{du'}{dy'} \left(h' + \frac{k-1}{2} \text{Ma}_\infty^2 u'^2 \right) + \left(\frac{1}{\sigma} - 1 \right) \left(\mu' \frac{dh'}{dy'} \right) = A_2 \tag{15}$$

Since $h = c_P T$ ..., then in dimensionless form

$$h' = T'. \tag{16}$$

Let us substitute (16) into (15) and subtract the equal terms. We obtain the following equation:

$$\mu' \left(k-1 \right) \text{Ma}_\infty^2 u' \frac{du'}{dy'} + \frac{1}{\sigma} \left(\mu' \frac{dT'}{dy'} \right) = A_2 \tag{17}$$

Equation (17) can be written in a simpler form if equation (14) is used:

$$\left(\tau_0 \left(k-1 \right) \text{Ma}_\infty^2 u' - A_2 \right) \frac{du'}{dy'} + \frac{\tau_0}{\sigma} \frac{dT'}{dy'} = 0 \tag{18}$$

The system of equations (3), (14), and (18) allows for obtaining a numerical solution to the problem for the unknown temperature and velocity functions. Indeed, we have the following system of equations:

$$\frac{du'}{dy'} = \frac{\tau_0}{(T')^{3/2} \left(\frac{1+T^*}{T'+T^*} \right)}, \quad \left(\tau_0 \left(k-1 \right) \text{Ma}_\infty^2 u' - A_2 \right) \frac{du'}{dy'} + \frac{\tau_0}{\sigma} \frac{dT'}{dy'} = 0 \tag{19}$$

The system matrix (19) has the following form:

$$\begin{pmatrix} 1 & 0 \\ \left(\tau_0 \left(k-1 \right) \text{Ma}_\infty^2 u' - A_2 \right) & \frac{\tau_0}{\sigma} \end{pmatrix} = \begin{pmatrix} \frac{\tau_0}{(T')^{3/2} \left(\frac{1+T^*}{T'+T^*} \right)} \\ 0 \end{pmatrix}$$

$$\frac{du'}{dy'} = \frac{\tau_0}{(T')^{3/2} \left(\frac{1+T^*}{T'+T^*} \right)}, \quad \frac{dT'}{dy'} = -\frac{\sigma \left(\tau_0 \left(k-1 \right) \text{Ma}_\infty^2 u' - A_2 \right)}{(T')^{3/2} \left(\frac{1+T^*}{T'+T^*} \right)} \tag{20}$$

As an example, a temperature of -53°C at infinity was specified for the calculations. Then, taking into account Sutherland's constant, equation (20) takes the following working form:

$$\frac{du'}{dy'} = \frac{-1}{(T')^{3/2} \left(\frac{1+0.5471}{T'+0.5471} \right)}, \quad \frac{dT'}{dy'} = \frac{0.7 \left(0.4 \text{Ma}_\infty^2 u' + A_2 \right)}{(T')^{3/2} \left(\frac{1+0.5471}{T'+0.5471} \right)} \tag{21}$$

The constant A_2 was determined from the boundary condition:

$$u' \Big|_{y'=y'_{BL}} = 0$$

It is worth noting that at relatively low Mach numbers, a decrease in wall temperature is observed, whereas at higher values, the opposite occurs. This correlates with the work of von Kármán [7].

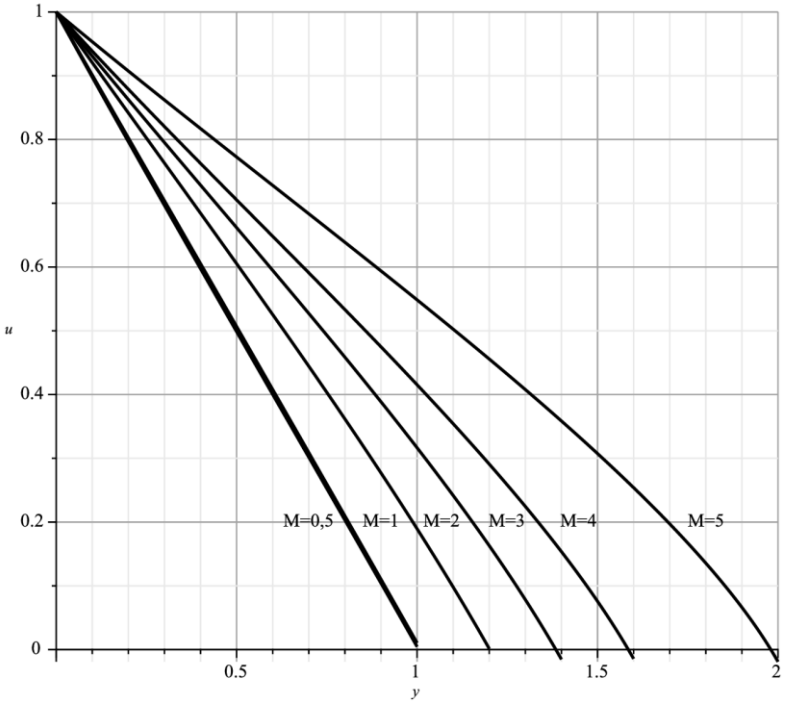


Fig. 3. Distributions of dimensionless velocity values for Mach numbers 0.5, 1, 2, 3, 4, and 5.

The question arises: which is correct—assuming the dimensionless thickness of the domain is 1, or extending it to 2? After all, this leads to vastly different surface temperature values. The answer to this question can be found in the work of Takhar [15]—where the boundary layer domain extends to several units of the dimensionless coordinate—as well as in the work of von Kármán [7]. Experimental data [10] also indicate that the asymptotic value is reached at approximately ten units of the dimensionless coordinate.

Let us now consider the central question of this study regarding the inverse problem: is the flow region in the immediate vicinity of a solid wall identical—in terms of wall friction—for boundary layers with and without a pressure gradient? Asymptotic analysis has shown that, for small values of the dimensionless (transverse) coordinate, the velocity field for the body motion problem (a zero-pressure-gradient boundary layer) is indeed a linear function. Thus, no analogy—or reversibility—is observed: while the velocity in a gradient boundary layer exhibits a non-linear dependence on the coordinate as it approaches the asymptotic limit (the outer edge of the boundary layer), a virtually linear dependence is observed for pressure gradient free flow across all Mach number values. It is impossible to "map" the velocity field function from the pressure gradient flow problem to the corresponding pressure gradient free flow problem. This is entirely logical, given that the problems are described by different systems of equations; each system contains an additional term accounting for the non-zero flow longitudinal gradient.

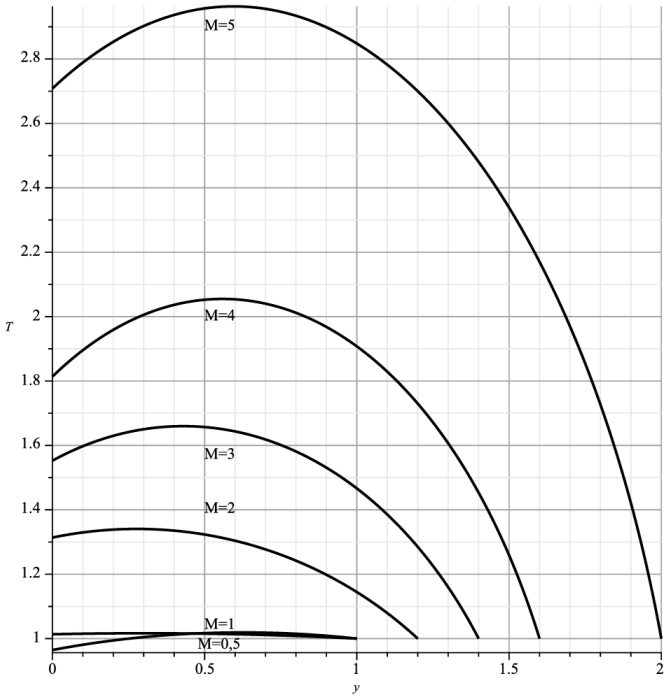


Fig. 4. Distributions of dimensionless temperature values for Mach numbers 0.5, 1, 2, 3, 4, and 5.

To answer this question, let us consider the asymptotic behavior of both flows near a solid wall. To this end, we approximate the temperature field using a Taylor power series:

$$T' = T'_W + a_1 y' + a_2 y'^2 \tag{22}$$

The first derivatives of expressions (21) and (22) are as follows:

$$\frac{dT'}{dy'}\Big|_{y'=0} \approx a_1 \tag{23}$$

Since the second derivative of the temperature function appears in equation (2), we also find an approximation for it:

$$\frac{d^2T'}{dy'^2}\Big|_{y'=0} \approx 2a_2 \tag{24}$$

Substitute expressions (23)–(24) into equation (2). We obtain:

$$0 = (k-1)\mathrm{Ma}_\infty^2 \mu' \left(\frac{du'}{dy'}\right)^2 + \frac{1}{\sigma}\left(\frac{d\mu'}{dy'}a_1 + \mu' 2a_2\right) \tag{25}$$

Taking (14) into account, equation (25) takes the following form:

$$0 = (k-1)\mathrm{Ma}_\infty^2 \frac{\tau_0^2}{\mu'} + \frac{1}{\sigma}\left(\frac{d\mu'}{dy'}a_1 + \mu' 2a_2\right) \tag{26}$$

Let us multiply (26) by the viscosity and rewrite it in the form:

$$\frac{a_1}{2}\frac{d\mu'^2}{dy'} = -\left[\sigma(k-1)\mathrm{Ma}_\infty^2\tau_0^2 + 2a_2\,\mu'^2\right] \tag{27}$$

Equation (27) can be presented in a simple form:

$$\frac{dM'}{dy'} = B_1 + B_2 M' \tag{28}$$

with $M' = \mu'^2$

The general solution (28) is as follows:

$$M' = \mu'^2 = -\frac{B_1}{B_2} + C_1 \exp\left(B_2 y'\right) \tag{29}$$

Or

$$\mu' = \left(-\frac{B_1}{B_2} + C_1 \exp\left(B_2 y'\right)\right)^{1/2} \tag{30}$$

Substituting (30) into (14), we obtain

$$\frac{du'}{dy'} = \frac{\tau_0}{\left(-\frac{B_1}{B_2} + C_1 \exp\left(B_2 y'\right)\right)^{1/2}} \tag{31}$$

The general solution of equation (31) is as follows:

$$u' = \frac{2\tau_0}{\sqrt{B_1 B_2}}\, arctg\left(\frac{B_2\sqrt{C_1 \exp(B_2 y') - B_1 / B_2}}{\sqrt{B_1 B_2}}\right) + C_2 \tag{32}$$

After meeting the boundary condition for (32) and setting the remaining constants to unity, we have:

$$u' = 1 + 2\left(arctg\left(\sqrt{C_1 \exp(y') - 1}\right) - arctg\left(\sqrt{C_1 - 1}\right)\right) \tag{33}$$

For value $C_1 = 2$ of the single constant in (33), the velocity graph is practically linear. Thus, within the framework of the problem of a plane moving in a still fluid, it is impossible to obtain the result found for the problem of an immobile plane in a flowing fluid.

Conclusions. A driving force is required to generate fluid flow. In nature, such forces include gravity and associated forces (such as the Archimedes or buoyancy force). Furthermore, temperature non-uniformity results in pressure gradients, which in turn generate flows. In the context of aviation, flow within the boundary layer on a plate can arise in only two ways. The first (the direct problem) is a pressure gradient free layer formed solely by the friction between a moving body and the fluid. The second approach (the inverse problem) involves a gradient flow artificially generated in a wind tunnel. The presence of a longitudinal pressure gradient alters the kinematics of the velocity field; consequently, distinct velocity field distributions arise in gradient and gradient free boundary layers. The numerical results obtained have been compared with those from relevant studies.

These results demonstrate that the principle of flow reversibility (or the reversibility of motion as a whole) cannot be applied.

Relevant unsteady problems could be considered for future research.

References

1. Lukianov P.V., Song L. Optimal Character and Different Nature of Flows in Laminar Boundary Layers of incompressible Fluid Flow Problems of Friction and Wear. 2022. № 4(97). P. 52-60.

2. Song L., Lukianov P.V., Badakh V.M., Tarasenko T.V. The effect of partial slip on the surface pressure distribution in a slightly compressible flow development region in the boundary layer. Problems of Friction and Wear. 2024. №3(104). P. 54-64.

3. Lukianov Pavlo, Song Lin. Unsteady Incompressible Laminar Boundary Layer: Time and Space Variable Molecular Viscosity. Авіаційно-Космічна Техніка і Технологія. 2023. № 3(187). С. 50—60.

4. Lukianov Pavlo, Song Lin. Flow development region in the boundary layer: two-component molecular viscosity and partial slip. Авіаційно-Космічна Техніка і Технологія. 2023. № 6(192). С. 38—47.

5. Schlichting H. Boundary layer theory. / H. Schlichting. – New York: McGraw-Hill, seventh ed., 1979.

6. Stokes G.G. On the theories of the internal friction of fluids in motion, and the equilibrium and motion of elastic solids. Trans. Cambridge Philos. Soc. 1845. Vol. 8. P. 287-305.

7. Th. Von Karman and H.S. Tsien. Boundary layer in Compressible Fluids. Journal of Aeronautical Sciences.DOI:10.2514/8.591.

8. Loitsianski L.G. The development of boundary layer theory in the USSR . Ann. Rev. fluid. Mech. 1970. Vol. 2. P. 1--15.

9. Blasius H. Grenzschichten in Flussigkeiten mit kleiner Reibung / Y. Blasius// Z. Math. Phys. 1908, v. 56, p. 1-37.

10. O'Donnell R.M. Experimental investigation at Mach number of 2.41 of average skin friction coefficients and velocity profiles for laminar and turbulent boundary layers and an assessment of probe effects. NACA Technical notes 3122. Washington, 1954.

11. Kenichi- Kusukawa, Shigeaki Suwa, Takeo Nakagawa. An interation method to solve the boundary layer flow past flat plate. Journal of applied mathematics and physics. 2014, 2, 35-40. http://dx.doi.org/10.4236/jamp.2014.24005

12. Howarth L. On the solution of laminar boundary layer equations. Proceeding of the Royal Society of London. Series A, Mathematics and Physics Sciences. V. 164 Pp. 547-579. http://dx.doi.org/10.1098/rspa.1938.0037.

13. Taniguchi N., Ohmichi Yu., Suzuki K. Nonlinear optimal forcing analysis on subsonic flow around an airfoil. J. Fluid Mech. (2026), vol. 1031, A1, http://doi:10.1017/jfm.2026.11280

14. Ackroyd J.A.D. On the laminar compressible boundary layer with stationary origin on a moving flat wall. Proc. Camb. Phil. Soc. 1967. V. 3. pp. 871-888.

15. Takhar H.S., Nitu S. and Pop I. Boundary layer flow due to a moving plate: variable fluid properties. Acta Mechanica. 1991, v. 90, pp. 37-42.

16. L. O. F. JEROMIN, Dipl-Ing. An Experimental Investigation of the Compressible Turbulent Boundary Layer with Air injection Reports and Memoranda No. 3526* November 1966

17. Wendt V., Simen M., Hanifi A. An experimental and theoretical investigation of instabilities in hypersonic flat plate boundary layer flow. Physics of Fluids. 1995. Vol. 7, No. 4. P. 877–887. DOI: 10.1063/1.868610.

18. Pfenninger W. Boundary layer suction experiments with laminar flow at high Reynolds numbers in the inlet length of a tube by various suction methods. Boundary Layer and Flow Control / ed. by G. V. Lachmann. Pergamon Press, 1961. Vol. 2. P. 961–980.

19. W.W. Lee and R.T. Davis, Laminar boundary layers on moving continuous surfaces, Chemical Engineering Science, 1972, Vol. 27, pp. 2 129-2149.

20. Wen-Jei Yang, Harng-Sen Huang, Unsteady Compressible Laminar Boundary-Layer Flow over a Flat Plate, 1969, AIAA JOURNAL, VOL. 7, NO. 1.

21. J.A.D. Ackroyd, On the laminar compressible boundary layer with stationary origin on a moving flat wall, 1967, Proc. Camb. Phil. Soc., 63, 871.

22. Hang Xu, Shi-Jun Liao, Laminar flow and heat transfer in the boundary-layer of non-Newtonian fluids over a stretching flat sheet, 2009, Computers and Mathematics with Applications 57 1425–1431, https://doi.org/10.1016/j.camwa.2009.01.029.

23. Damien Biau, Laminar-turbulent separatrix in a boundary layer flow, 2012, Physics of Fluids (1994-present) 24, 034107; doi: 10.1063/1.3696303.

24. Hanin M. On Rayleigh's problem for compressible fluids. *Quart. Journ. Mech. and Applied Math*. 1960. Vol. XIII, Pt. 2. P. 185–198.

25. Howarth L. Some aspects of Rayleigh's problem for a compressible fluid. *Quart. Journ. Mech. and Applied Math*. 1951. Vol. IV, Pt. 2. P. 157–169.

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Pavlo Lukianov – Ph.D., Associate Professor in Aviation Engineering Department, National University “Kyiv Aviation Institute”, 03058, Ukraine, Kyiv, Lubomyr Husar ave., 1, tel. number: + 38 096 125 47 22, E-mail: Pavel_Lukianov@ukr.net, ORCID:https://orcid.org/0000-0002-5043-6182

Utiuhov Yeugen Viktorovych – postgraduate in Aviation Engineering Department, National University “Kyiv Aviation Institute”, 03058, Ukraine, Kyiv, Lubomyr Husar ave., 1, tel. number: +380 68 615 7694, zytygov@gmail.com, https://orcid.org/0009-0002-1540-4000/

Лук’янов Павло Володимирович – канд. фіз.-мат. наук, старший науковий співробітник, доцент кафедри авіаційної інженерії, Національний університет «Київський авіаційний інститут», пр. Любомира Гузара, 1, м. Київ, Україна, 03058, тел.: +380 96 125 4722, pavel_lukianov@ukr.net, https://orcid.org/0000-0002-5043-6182.

Утюгов Євген Вікторович - аспірант кафедри авіаційной інженерії, Національний університет «Київський авіаційний інститут», пр. Любомира Гузара, 1, м. Київ, Україна, 03058, +380 68 615 7694, zytygov@gmail.com, https://orcid.org/0009-0002-1540-4000/

П. В. ЛУК'ЯНОВ, Є. В. УТЮГОВ

ВІДСУТНІСТЬ ФІЗИЧНОЇ ВІДПОВІДНОСТІ ТА ОБЕРНЕНОСТІ ЛАМІНАРНИХ СТИСЛИВИХ ГРАДІЄНТНОЇ ТА БЕЗ-ГРАДІЄНТНОЇ ТЕЧІЙ ПРИМЕЖОВОГО ШАРУ

В роботі розглянуто питання фізичної відповідності існуючих теорій примежового ламінарного шару стисливої течії рідини. Вказано, що в класичних роботах (Блазіуса, Кармана) рівняння, що описують рух рідини, відповідають прямій задачі руху тіла у нерухомій на нескінченості рідині, утім граничні умови, що використовуються, відповідають оберненій задачі обтікання рідиною нерухомого тіла (течія у аеродинамічній трубі). Таким чином, наявна фізична невідповідність теорій реальним течіям. Показано, що повна оберненість задач є неможливою, оскільки неможна результати однієї течії використати для опису іншої через різну природу (динаміку) і кінематику цих течій. Наведено фізичні постановки прямої (рух тіла в рідині) та оберненої (обтікання рідиною нерухомого тіла) задач та їх чисельні розв'язки. Для оберненої задачі поле швидкості порівнюється із відомими результатами (Кармана), а для прямої задачі – із іншими роботами. В цілому в обидвох задачах якісно результати є схожими із тими, що порівнюються.

Ключові слова: примежовий шар, ламінарна течія, стислива течія, градієнтна та без градієнтна течії, обтікання, рівняння Нав'є-Стокса.

References

1. Lukianov P.V., Song L. Optimal Character and Different Nature of Flows in Laminar Boundary Layers of incompressible Fluid Flow Problems of Friction and Wear. 2022. № 4(97). P. 52-60.

2. Song L., Lukianov P.V., Badakh V.M., Tarasenko T.V. The effect of partial slip on the surface pressure distribution in a slightly compressible flow development region in the boundary layer. Problems of Friction and Wear. 2024. №3(104). P. 54-64.

3. Lukianov Pavlo, Song Lin. Unsteady Incompressible Laminar Boundary Layer: Time and Space Variable Molecular Viscosity. Авіаційно-Космічна Техніка і Технологія. 2023. № 3(187). С. 50—60.

4. Lukianov Pavlo, Song Lin. Flow development region in the boundary layer: two-component molecular viscosity and partial slip. Авіаційно-Космічна Техніка і Технологія. 2023. № 6(192). С. 38—47.

5. Schlichting H. Boundary layer theory. / H. Schlichting. – New York: McGraw-Hill, seventh ed., 1979.

6. Stokes G.G. On the theories of the internal friction of fluids in motion, and the equilibrium and motion of elastic solids. Trans. Cambridge Philos. Soc. 1845. Vol. 8. P. 287-305.

7. Th. Von Karman and H.S. Tsien. Boundary layer in Compressible Fluids. Journal of Aeronautical Sciences.DOI:10.2514/8.591.

8. Loitsianski L.G. The development of boundary layer theory in the USSR . Ann. Rev. fluid. Mech. 1970. Vol. 2. P. 1--15.

9. Blasius H. Grenzschichten in Flussigkeiten mit kleiner Reibung / Y. Blasius// Z. Math. Phys. 1908, v. 56, p. 1-37.

10. O'Donnell R.M. Experimental investigation at Mach number of 2.41 of average skin friction coefficients and velocity profiles for laminar and turbulent boundary layers and an assessment of probe effects. NACA Technical notes 3122. Washington, 1954.

11. Kenichi- Kusukawa, Shigeaki Suwa, Takeo Nakagawa. An interation method to solve the boundary layer flow past flat plate. Journal of applied mathematics and physics. 2014, 2, 35-40. http://dx.doi.org/10.4236/jamp.2014.24005

12. Howarth L. On the solution of laminar boundary layer equations. Proceeding of the Royal Society of London. Series A, Mathematics and Physics Sciences. V. 164 Pp. 547-579. http://dx.doi.org/10.1098/rspa.1938.0037.

13. Taniguchi N., Ohmichi Yu., Suzuki K. Nonlinear optimal forcing analysis on subsonic flow around an airfoil. J. Fluid Mech. (2026), vol. 1031, A1, http://doi:10.1017/jfm.2026.11280

14. Ackroyd J.A.D. On the laminar compressible boundary layer with stationary origin on a moving flat wall. Proc. Camb. Phil. Soc. 1967. V. 3. pp. 871-888.

15. Takhar H.S., Nitu S. and Pop I. Boundary layer flow due to a moving plate: variable fluid properties. Acta Mechanica. 1991, v. 90, pp. 37-42.

16. L. O. F. JEROMIN, Dipl-Ing. An Experimental Investigation of the Compressible Turbulent Boundary Layer with Air injection Reports and Memoranda No. 3526* November 1966

17. Wendt V., Simen M., Hanifi A. An experimental and theoretical investigation of instabilities in hypersonic flat plate boundary layer flow. Physics of Fluids. 1995. Vol. 7, No. 4. P. 877–887. DOI: 10.1063/1.868610.

18. Pfenninger W. Boundary layer suction experiments with laminar flow at high Reynolds numbers in the inlet length of a tube by various suction methods. Boundary Layer and Flow Control / ed. by G. V. Lachmann. Pergamon Press, 1961. Vol. 2. P. 961–980.

19. W.W. Lee and R.T. Davis, Laminar boundary layers on moving continuous surfaces, Chemical Engineering Science, 1972, Vol. 27, pp. 2 129-2149.

20. Wen-Jei Yang, Harng-Sen Huang, Unsteady Compressible Laminar Boundary-Layer Flow over a Flat Plate, 1969, AIAA JOURNAL, VOL. 7, NO. 1.

21. J.A.D. Ackroyd, On the laminar compressible boundary layer with stationary origin on a moving flat wall, 1967, Proc. Camb. Phil. Soc., 63, 871.

22. Hang Xu, Shi-Jun Liao, Laminar flow and heat transfer in the boundary-layer of non-Newtonian fluids over a stretching flat sheet, 2009, Computers and Mathematics with Applications 57 1425–1431, https://doi.org/10.1016/j.camwa.2009.01.029.

23. Damien Biau, Laminar-turbulent separatrix in a boundary layer flow, 2012, Physics of Fluids (1994-present) 24, 034107; doi: 10.1063/1.3696303.

24. Hanin M. On Rayleigh's problem for compressible fluids. *Quart. Journ. Mech. and Applied Math*. 1960. Vol. XIII, Pt. 2. P. 185–198.

25. Howarth L. Some aspects of Rayleigh's problem for a compressible fluid. *Quart. Journ. Mech. and Applied Math*. 1951. Vol. IV, Pt. 2. P. 157–169.