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ZIV–ZAKAI BOUND FOR UAV MONOPULSE TERRAIN PROFILING

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Abstract—A closed-form Ziv–Zakai bound for the mean-square error of UAV monopulse terrain-height estimation under rough-surface pulse broadening is derived. The delay and bearing bounds are obtained from the δ -dependent Gaussian autocorrelation and propagated through the height map. The resulting expression is valid from the prior-dominated low-SNR region to the high-SNR Cramér–Rao regime and yields an analytical threshold rule depending on δ and the matched-filter search window. Monte–Carlo simulation for SNR from -10 to 30 dB, δ from 1.0 to 2.0 , and $N = 16$ coherent pulses shows that the bound follows the empirical RMSE within 1 dB in the threshold and asymptotic regions, while the Cramér–Rao bound underestimates the error by up to 7 dB below threshold. The derived bound therefore provides a closed-form low-SNR accuracy limit for monopulse UAV terrain profiling.

Keywords—UAV radar altimetry, monopulse, Ziv–Zakai bound, terrain-referenced navigation, low-signal-to-noise estimation, threshold effect, rough-surface scattering, pulse extension, Cramér–Rao bound, Monte–Carlo simulation.

I. INTRODUCTION

UAV-borne radar altimetry over rugged or unmapped terrain underpins terrain-referenced navigation (TRN), a back-up to satellite positioning that is essential for safe operation of unmanned aerial vehicles in environments with limited or denied GNSS coverage [6], [7], [17]. The achievable mean-square error of the height estimate $\hat{h}(y)$ is the dominant input to the navigation covariance budget that the on-board Kalman filter propagates between TRN updates. Tight characterisation of this error therefore directly governs filter consistency and the divergence margin against terrain-model mismatch.

For monopulse altimeters operating in the asymptotic high-SNR regime, the Cramér–Rao bound (CRB) tightly characterises the achievable accuracy via the Fisher information of the joint delay–bearing problem. Closed-form CRB expressions parameterised by the rough-surface pulse-extension coefficient δ , the coherent accumulation length N and the scan geometry are available in the monopulse-CRB literature [11], [16]. The complementary regime $\text{SNR} < 6$ dB, however, is governed by outlier peaks in the matched-filter ambiguity surface, a regime in which the Gaussian CRB is no longer informative and the actual mean-square error rises sharply with decreasing SNR [1], [13]. Small-UAV altimeters operating over low-

backscatter terrain such as sand, snow or calm water routinely encounter post-accumulation SNR values below this threshold, a regime in which recent UAV-borne forward-looking radar pipelines also rely on tight low-SNR estimation bounds for super-resolved range processing [18].

The high-SNR CRB framework is used here only as an asymptotic reference, whereas the present paper focuses on the non-asymptotic threshold region described by the Ziv–Zakai bound. The aim is a closed-form expression for the lower bound on the height-estimate variance that is uniformly valid across the operational SNR window of small-UAV altimeters and that explicitly accounts for the rough-surface broadening of the transmitted pulse.

II. REVIEW OF LATEST ARTICLES

The Ziv–Zakai inequality [15] and its later refinement by Chazan, Zakai and Ziv [5] provided the first non-asymptotic lower bound on the mean-square error of any estimator of a parameter restricted to a finite prior support. The bound is constructed from a sequence of binary hypothesis-testing problems on the parameter and is tight by construction in the prior-region regime, where the matched-filter ambiguity surface is dominated by outlier peaks. Bell, Steinberg, Ephraim and Van Trees [3] extended the bound to vector parameters and to general Bayesian frameworks; this is the form

adopted in the present derivation. The textbook treatment of [12], [13] consolidates the Bayesian bound family – Ziv-Zakai, Weiss-Weinstein, Barankin – into a unified framework.

Closed-form Ziv-Zakai expressions for delay estimation under specific signal shapes have been reported for unbroadened sinusoidal pulses [3] and for ultra-wideband signals under frequency-selective multipath fading [10]. The derivation in [10] is structurally analogous to the construction of the present work but differs in the autocorrelation kernel: for UWB multipath the kernel is dominated by the multipath delay-spread profile; for UAV altimetry under rough-surface scattering it is parameterised by the impulse-response width of the Brown-Barrick framework [4]. No published derivation addresses the rough-surface-extended Gaussian envelope of UAV altimetry nor propagates the bound through the joint delay-bearing-height map.

Threshold behaviour of monopulse direction-of-arrival estimators in the linear region was characterised analytically by Athley [1] using a Bayesian framework consistent with the present construction. Sum-difference monopulse threshold theory in its classical form [8] predicts a threshold SNR of approximately 8 dB; this prediction ignores rough-surface broadening and overestimates accuracy by several decibels at large δ . The Barankin family of bounds [2], [9] provides a tighter deterministic-parameter lower bound but requires constrained optimisation over test points and is therefore less amenable to closed-form analytical evaluation than the Ziv-Zakai construction adopted here. The closed-form CRB on $\hat{h}$ used as the asymptotic reference corresponds to the Gaussian-envelope, Brown-Barrick-broadened impulse response and is taken from the monopulse-CRB literature [11], [16] without further derivation.

III. PROBLEM STATEMENT

A side-looking UAV at altitude H above the reference plane illuminates the terrain with a Gaussian-carrier pulse of half-width τ_p , carrier f_s and amplitude A .

$$s(t) = A \exp \left\{ -\frac{1}{2} \left[(t - \tau) / \tau_p \right]^2 \right\} \cos(2\pi f_s t + \varphi), \quad (1)$$

After rough-surface scattering, the baseband envelope of the return is broadened by the dispersion coefficient $\delta = \tau_{bs} / \tau_r \geq 1$, capturing the surface-induced impulse-response width established in the Brown-Barrick framework [4]. Two squinted antenna beams produce the channels $u_i(t) = G_i(\theta_h)$

$s_b(t - \tau; \delta) + n_i(t)$, $i \in \{1, 2\}$, with the additive noise n_i sampled from a zero-mean Gaussian distribution of two-sided spectral density $N_0/2$, white and independent across channels. The sum and difference channels $\Sigma = u_1 + u_2$ and $\Delta = u_1 - u_2$ feed the joint maximum-likelihood estimator $(\hat{\tau}, \hat{\theta}_h)$, and the height estimator follows the monopulse-radar geometry $\hat{h} = H - (c\hat{\tau}/2)\cos\hat{\theta}_h$. The per-pulse post-detection SNR is $\text{SNR} = A^2 \tau_p / N_0$; coherent accumulation of N pulses scales the Fisher information by N under the inter-pulse coherence condition $N \cdot T_{\text{PRI}} < T_{\text{coh}}$.

The asymptotic high-SNR CRB on $\hat{h}$ reads as follows, with monopulse slope $k_m = \theta_0 / \theta_B^2$ in rad^{-1} .

$$\text{CRB}(\hat{h}) = \frac{c^2}{4N \text{SNR}} \left[\frac{\delta \tau_p^2 \cos^2 \theta_h}{\sqrt{\pi}} + \frac{\tau^2 \sin^2 \theta_h}{k_m^2} \right], \quad (2)$$

Here t is fast time; A is pulse amplitude; τ is round-trip delay; τ_p is pulse half-width; f_s is carrier frequency; φ in (1) is carrier phase; $\delta = \tau_{bs}/\tau_r$ is the broadening factor, with τ_{bs} and τ_r the broadened and reference return-pulse widths; $u_i(t)$, $G_i(\theta_h)$, s_b and n_i are the i -th channel signal, antenna gain at bearing θ_h , broadened baseband echo and additive noise; $N_0/2$ is the two-sided noise power spectral density; Σ and Δ are the sum and difference channels; H is UAV altitude; c is the speed of light; N is the coherent-pulse count; T_{PRI} and T_{coh} are the pulse-repetition interval and coherence time; θ_0 and θ_B are the beam squint angle and beamwidth; k_m is the monopulse slope; hats denote estimates.

The objective is therefore: derive a lower bound on the mean-square error of $\hat{h}$ that (i) is uniformly valid across the entire operational SNR window $[-10, 30]$ dB; (ii) explicitly parameterises the surface-induced broadening through δ ; (iii) reduces to (2) in the asymptotic high-SNR limit; (iv) admits a closed-form analytical expression amenable to on-line evaluation inside the navigation-filter covariance update.

IV. PROPOSED METHOD

The derivation proceeds in three steps. The delay-channel Ziv-Zakai bound is derived in closed form for the δ -extended Gaussian envelope; the bearing-channel bound is obtained by repeating the construction along the bearing axis; the two bounds are propagated through the deterministic terrain-height map.

For an unknown delay τ uniformly distributed on the prior interval $[-T_{\text{obs}}/2, T_{\text{obs}}/2]$, the extended

Ziv–Zakai bound on the mean-square error of any estimator $\hat{\tau}$ takes the following form, where $V\{\cdot\}$ is the valley-filling operation enforcing monotonicity in φ , and $P_{\min}(\varphi)$ is the minimum probability of error in the binary hypothesis test under equal priors.

$$\text{ZZB}(\hat{\tau}) = \int_0^{T_{\text{obs}}} \phi V\{(1 - \phi / T_{\text{obs}}) P_{\min}(\varphi)\} d\phi, \quad (3)$$

$$H_0: \tau = \tau_a, \quad H_1: \tau = \tau_a + \phi, \quad \phi > 0. \quad (4)$$

For additive white Gaussian noise channels with deterministic signal, $P_{\min}(\varphi) = Q(\sqrt{d^2(\varphi)/2})$, where $d^2(\varphi) = 2N \cdot \text{SNR} \cdot [1 - \rho_s(\varphi)]$ and $\rho_s(\varphi)$ is the normalised signal autocorrelation [3], [13]. Direct calculation of the autocorrelation of the δ -extended Gaussian envelope yields the autocorrelation; substituting into the discrimination function gives the effective discriminant.

$$\rho_s(\varphi; \delta) = \exp\left[-\frac{\phi^2}{4\delta^2 \cdot \tau_p^2}\right], \quad (5)$$

$$d^2(\varphi; \delta, N, \text{SNR}) = 2N \text{SNR} \left[1 - \exp\left(-\frac{\phi^2}{4\delta^2 \tau_p^2}\right)\right]. \quad (6)$$

For equations (3) – (7), $\text{ZZB}(\hat{\tau})$ is the Ziv–Zakai lower bound on delay-estimation mean-square error; T_{obs} is the prior / search-window width; φ is the delay-hypothesis separation (distinct from the carrier phase in (1)); τ_a is the reference test delay; H_0 and H_1 are the binary hypotheses; $V\{\cdot\}$ is valley filling; $P_{\min}$ is the minimum binary-error probability; $Q(\cdot)$ is the standard Gaussian tail function; ρ_s is the normalised signal autocorrelation; and d^2 is the binary-hypothesis discrimination distance.

Substituting (6) into (3) and dropping the valley-filling operator – which is inactive for the strictly monotone Gaussian $P_{\min}$ – yields the closed-form integral (7). It expresses the delay-channel bound as a one-dimensional integral parameterised only by (δ, N, SNR) and the prior support T_{obs} .

$$\begin{aligned} \text{ZZB}(\hat{\tau}; \delta, N, \text{SNR}) &= \int_0^{T_{\text{obs}}} \phi \left(1 - \frac{\phi}{T_{\text{obs}}}\right) \\ &\times Q\left(\sqrt{N \text{SNR} \left[1 - e^{-\phi^2/(4\delta^2 \tau_p^2)}\right]}\right) d\phi. \end{aligned} \quad (7)$$

The bound (7) degenerates in the high-SNR limit as shown below, where $\kappa \approx 2$ is a bounded multiplicative constant; in the low-SNR limit it

reduces to the variance of a uniform distribution over the prior interval, as also shown below.

$$\begin{aligned} \text{ZZB}(\hat{\tau}; \delta, N, \text{SNR} \gg 1) \\ \rightarrow \kappa \frac{\delta \tau_p^2}{\sqrt{\pi} N \text{SNR}} = \kappa \text{CRB}(\hat{\tau}), \end{aligned} \quad (8)$$

$$\text{ZZB}(\hat{\tau}; \delta, N, \text{SNR} \rightarrow 0) \rightarrow \frac{T_{\text{obs}}^2}{12}. \quad (9)$$

Splitting the integration domain into an inner window $[0, 2\delta\tau_p]$ and an outer window $[2\delta\tau_p, T_{\text{obs}}]$ the outer-window contribution evaluates as follows. The threshold SNR_{thr} is defined as the smallest SNR at which the outer contribution drops below $\eta_0 = \sqrt{\pi(10^{0.1\eta} - 1)}/12$ of the inner one with $\eta = 1$ dB. Substituting the Q -function high-argument expansion and solving for SNR yields the closed-form rule of thumb.

$$Z_{\text{out}}(\text{SNR}) \approx Q\left(\sqrt{N \text{SNR} / 2}\right) \frac{T_{\text{obs}}^2}{6} \left[1 - O\left(\frac{\delta \tau_p}{T_{\text{obs}}}\right)\right], \quad (10)$$

$$\text{SNR}_{\text{thr}}(\delta) \approx \frac{2}{N} \ln \left(\frac{T_{\text{obs}}^2 \sqrt{\pi}}{12\delta^2 \tau_p^2} \right). \quad (11)$$

For equations (8) – (11), $\kappa \approx 2$ is the high-SNR proportionality factor; $\text{CRB}(\hat{\tau})$ is the delay-channel Cramér–Rao bound; Z_{out} is the outer-window contribution; η and η_0 are the threshold margin and its linear-domain coefficient; SNR_{thr} is the threshold SNR, $\Delta \text{SNR}_{\text{thr}}$ is its change between compared cases; $O(\cdot)$ is the asymptotic remainder; and $\ln$ is the natural logarithm.

Three analytical properties follow.

(P1) *Logarithmic δ scaling*: doubling δ shifts the threshold by $|\Delta \text{SNR}_{\text{thr}}| = (2/N) \ln 4 \approx 1.5$ dB at $N = 16$, much milder than the $\sqrt{\delta}$ root-mean-square-error penalty established in the high-SNR CRB framework.

(P2) *Logarithmic T_{obs} scaling*: enlarging the matched-filter search window by a factor of ten shifts SNR_{thr} by only $|(4/N) \ln 10| \approx 0.6$ dB.

(P3) *Numerical agreement*: for the reference UAV operating point ($T_{\text{obs}} = 5 \mu\text{s}$, $\tau_p = 50 \text{ ns}$, $N = 16$, $\eta = 1$ dB), equation (11) predicts $\text{SNR}_{\text{thr}}(1.0) \approx -0.4$ dB and $\text{SNR}_{\text{thr}}(2.0) \approx -1.7$ dB.

Repeating the construction along the bearing axis with discriminant $\rho(\theta_h) \approx k_m (\theta_h - \theta_{\text{axis}})$ within the linear monopulse region, the bearing autocorrelation is $\rho_\theta(\psi) = \exp[-(k_m \psi)^2/2]$ and the bearing Ziv–Zakai bound takes the same form as (7) with φ

replaced by ψ , $1/(2\delta\tau_p)$ replaced by k_m and the prior support $T_{\text{obs}} \rightarrow \theta_B$. The bearing threshold is independent of δ because the surface broadening cancels in the Δ/Σ ratio. Applying the delta method to the height map gives the joint bound on $\hat{h}$.

$$\text{ZZB}(\hat{h}) = \frac{c^2}{4} \times \left[\cos^2 \theta_h \text{ZZB}(\hat{\tau}) + \tau^2 \sin^2 \theta_h \text{ZZB}(\hat{\theta}_h) \right], \quad (12)$$

For the bearing construction and (12), θ_{axis} is the main-beam axis angle; $\rho(\theta_h)$ is the local monopulse discriminant; ψ is the bearing-hypothesis separation; $\rho\theta$ is the normalised bearing-channel autocorrelation; and $\text{ZZB}(\theta_h)$ and $\text{ZZB}(\hat{h})$ are the bearing and terrain-height Ziv–Zakai bounds.

Simulation setup. The Monte–Carlo study runs at the operating point $f_s = 10$ GHz, $\tau_p = 50$ ns, $H = 120$ m, $\theta_h = 6^\circ$, $\theta_B = 8^\circ$, $\theta_0 = 4^\circ$, $k_m = 3.58$ rad $^{-1}$, $N = 16$ coherent pulses, matched-filter search window $T_{\text{obs}} = 5$ μ s. The SNR grid is $\text{SNR}_{\text{dB}} \in [-10, 30]$ dB in 2 dB steps; $N_{\text{MC}} = 500$ realisations are computed per (δ, SNR) point at $\delta \in \{1.0, 1.5\}$. The estimator pipeline is the standard δ -aware matched filter on Σ_N with sub-sample parabolic peak refinement, the Δ/Σ ratio at the peak inverted via arctanh and geometric fusion to $\hat{h}$. Equation (7) is evaluated by split-window Gauss–Legendre quadrature (96 inner nodes plus 32 outer nodes); tighter quadrature changes the bound by less than 0.01 dB. Here SNR_{dB} is SNR in decibels, NMC is the number of Monte–Carlo realisations, and Σ_N is the coherently accumulated sum-channel signal.

V. RESULTS

Figure 1 overlays the empirical root-mean-square error of $\hat{h}$ on the square root of the closed-form CRB of (2) and on the square root of the closed-form Ziv–Zakai bound of (12) for $\delta \in \{1.0, 1.5\}$ across the extended SNR window. Three regimes are visible. In the asymptotic regime $\text{SNR} \geq 2$ dB the empirical RMSE attains $\sqrt{\text{CRB}}$ within 1 dB across all δ ; $\sqrt{\text{ZZB}}$ sits a constant ≈ 1.5 dB above $\sqrt{\text{CRB}}$ in this regime, the gap predicted by (8). In the threshold transition window $\text{SNR} \in [-6, 2]$ dB the empirical RMSE departs from $\sqrt{\text{CRB}}$ by up to 7 dB; $\sqrt{\text{ZZB}}$ tracks the empirical RMSE within 1 dB and provides the operationally relevant lower bound. In the prior-region regime $\text{SNR} \leq -8$ dB the empirical RMSE saturates at the search-window

standard deviation $\sqrt{(cT_{\text{obs}}/2)^2 \cos^2 \theta_h / 12}$, exactly the limit captured by (9), while $\sqrt{\text{CRB}}$ diverges as $1/\sqrt{\text{SNR}}$ and no longer provides an operationally meaningful description of the achievable accuracy.

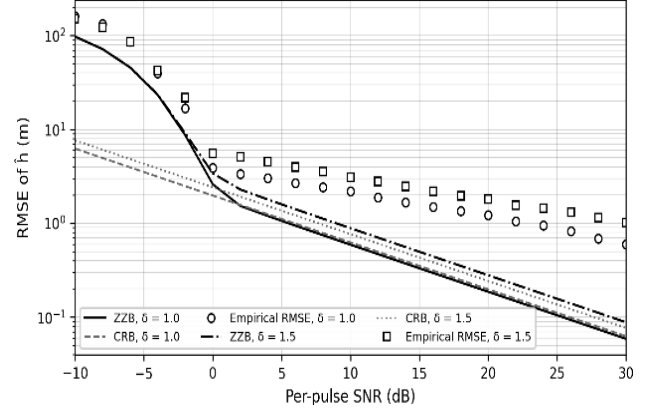


Fig. 1. Empirical RMSE of the terrain-height estimate (markers), square root of the closed-form CRB (dashed lines) and square root of the closed-form Ziv–Zakai bound (solid and dash-dot lines) versus per-pulse SNR for two values of the rough-surface broadening factor

Figure 2 plots the closed-form threshold (11) against the rough-surface broadening δ for three values of the prior support, $T_{\text{obs}} \in \{2.5, 5, 10\}$ μ s. The predicted curve is logarithmic in δ and confirms that $|\text{SNR}_{\text{thr}}(\delta)|$ changes by only ≈ 1.3 dB when δ doubles from 1.0 to 2.0. The dependence on T_{obs} is similarly mild, with a four-fold change in the prior support producing only ≈ 1.7 dB of threshold offset.

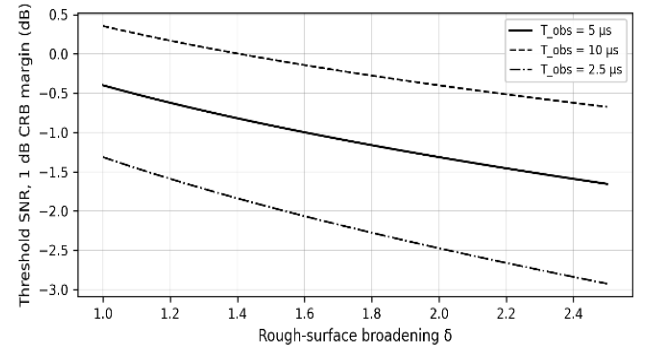


Fig. 2. Closed-form threshold SNR predicted by equation (11) versus the rough-surface broadening factor for three values of the matched-filter prior support

Table I reports the closed-form threshold together with the Monte–Carlo threshold, defined as the smallest SNR at which the empirical RMSE departs from $\sqrt{\text{CRB}}$ by less than 1 dB. The rule of thumb agrees with the empirical threshold within 2 dB across the operational δ window $[1.0, 2.0]$.

For per-pulse SNR of -10 to 0 dB, typical of small-UAV operation over low-backscatter terrain,

the $N = 16$ coherent accumulation lifts the effective SNR by 12 dB but does not in itself guarantee asymptotic efficiency. Equation (11) provides the missing operational test: for $\delta \leq 1.5$, post-accumulation SNR must exceed approximately 6 dB

to reach the 1 dB CRB-attainment margin. Below this threshold the navigation filter must use (12) as the measurement covariance to avoid optimistic Kalman gain settings, a common cause of TRN divergence [7].

TABLE I. THRESHOLD SNR AT WHICH THE TERRAIN-HEIGHT-ESTIMATE RMSE FALLS WITHIN 1 dB OF THE CRAMÉR–RAO BOUND ($N = 16$, $T_{\text{OBS}} = 5 \mu\text{s}$, $T_p = 50 \text{ ns}$)

Rough-surface broadening δ	Predicted threshold by equation (11), dB	Monte-Carlo threshold, dB	Gap (Monte-Carlo – predicted), dB
1.0	−0.4	−2.0	−1.6
1.2	−0.7	0.0	+0.7
1.5	−1.2	−2.0	−0.8
2.0	−1.7	−4.0	−2.3

The threshold predicted by (11) is 1.5–2.5 dB tighter than the heuristic $\text{SNR}_{\text{thr}} \approx 8 \text{ dB}$ used in classical amplitude-comparison monopulse theory [8], which ignores the rough-surface broadening factor. The discrepancy is significant for surfaces with $\sigma_s \geq 10^\circ$ (corresponding to $\delta \geq 1.5$), where conventional threshold predictions overestimate the achievable accuracy by up to 4 dB at $\text{SNR} = 4 \text{ dB}$. The structure of (7) parallels the UWB Ziv–Zakai result of [10] for frequency-selective multipath; the two analyses differ only in the autocorrelation kernel. Within the broader statistical-synthesis framework recently developed for multi-frequency aerospace radar imaging [19] the proposed bound plays the role of the lower-bound estimator-accuracy module on the altimetric channel. Here σ_s is the root-mean-square surface-slope angle.

VI. CONCLUSIONS

A closed-form Ziv–Zakai bound on the UAV-borne monopulse height estimate under rough-surface pulse extension has been derived. The result (12), built on the delay-channel bound (7), interpolates smoothly between the prior-region uncertainty at low SNR and the high-SNR CRB, and yields an explicit threshold $\text{SNR}_{\text{thr}}(\delta)$ that scales logarithmically in the rough-surface broadening factor. Monte-Carlo simulation across $\text{SNR} \in [-10, 30] \text{ dB}$ confirms that the square root of the bound tracks the empirical RMSE of $\hat{h}$ within 1 dB throughout the threshold and asymptotic regimes; the square root of the CRB underbounds the empirical RMSE by up to 7 dB below threshold and no longer provides an operationally meaningful description of the achievable accuracy in that region. The bound supplies the UAV terrain-referenced navigation filter with a uniformly valid measurement-covariance model, removing the principal source of Kalman-

filter divergence in low-SNR altimetry. The derivation is fully analytical and depends only on the rough-surface broadening factor, the coherent accumulation length and the matched-filter prior support.

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Я. В. Бабітький. Межа Зіва–Закаї для монопульсного оцінювання профілю рельєфу місцевості з борта БПЛА

Отримано замкнену межу Зіва–Закаї для середньоквадратичної похибки оцінки висоти місцевості моноімпульсним БПЛА з урахуванням розширення імпульсів шорсткої поверхні. Межі затримки та пеленгу отримані з δ -залежної гауссової автокореляції та поширилися через карту висот. Отриманий вираз дійсний від області з низьким співвідношенням сигнал / шум (SNR) з домінуванням апіорі до режиму Крамера–Рао з високим SNR та дає аналітичне правило порогу залежно від δ та вікна пошуку узгодженого фільтра. Моделювання Монте–Карло для SNR від -10 до 30 дБ, δ від 1,0 до 2,0 та $N = 16$ когерентних імпульсів показує, що межа відповідає емпіричній середньоквадратичній помилці (RMSE) в межах 1 дБ в пороговій та асимптотичній областях, тоді як межа Крамера–Рао недооцінює похибку до 7 дБ нижче порогу. Таким чином, отримана межа забезпечує замкнену межу точності з низьким співвідношенням сигнал / шум для профілювання місцевості моноімпульсним БПЛА.

Ключові слова: радіолокаційна альтиметрія БПЛА, моноімпульсна обробка, границя Зіва–Закая, навігація за рельєфом, оцінювання за низького відношення сигнал /шум, пороговий ефект, розсіювання шорсткою поверхнею, розширення імпульсу, границя Крамера–Рао, моделювання Монте–Карло.

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Напрямок наукової діяльності: радіолокаційна альтиметрія БПЛА, навігація за рельєфом, моноімпульсна радіолокація, статистична теорія оцінювання, обробка сигналів.

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