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Algorithmic Approaches to the Synthesis of Mathematical Models for Simulation of Aerodynamic Heating in Complex Structures

Developed algorithmic methods for synthesizing mathematical models to simulate aerodynamic heating in complex structures. The proposed approaches enhance model accuracy and computational efficiency, accommodating gradient heat distributions in dielectric layers, improving the design of antenna systems in aerodynamically heated environments.

Introduction

As aerospace vehicles operate at higher speeds, aerodynamic heating becomes a critical factor affecting the structural integrity and performance of onboard systems. Antenna fairings, which protect communication and navigation equipment, are particularly susceptible to temperature-induced changes in their dielectric properties. Accurate simulation of aerodynamic heating is essential for designing these components to ensure reliability and performance under extreme conditions.

Problem Statement

The primary challenge lies in accurately modeling the temperature gradients within complex structures, such as the dielectric layers of antenna fairings, during aerodynamic heating [1]. Traditional models often assume uniform temperature distributions or rely on simplifications that do not capture real thermal behavior. This leads to inaccuracies in predicting the electrical characteristics of dielectric materials, potentially compromising system performance [3].

Methods

Our approach focuses on developing algorithmic methods to synthesize mathematical models that accurately simulate aerodynamic heating in complex structures. [2] The methodology comprises several key components:

Advanced Heat Transfer Modeling

The heat transfer within the dielectric layer is governed by the three-dimensional transient heat conduction equation:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q \tag{1}$$

where:

- ρ is the density,
- c_p is the specific heat capacity,

- k is the thermal conductivity,
- T is the temperature,
- t is time,
- Q represents internal heat generation.

Algorithm Development

We developed algorithms based on:

- **Finite Element Method (FEM)**: To discretize the complex geometry of the structures and solve the heat conduction equation numerically.
- **Adaptive Mesh Refinement**: To enhance accuracy in regions with steep temperature gradients by refining the computational mesh where needed.

Material Property Variation

Incorporated temperature-dependent material properties into the models:

$$k(T), \quad c_p(T), \quad \rho(T) \tag{2}$$

This accounts for the changes in thermal conductivity, specific heat capacity, and density with temperature, which is critical for accurate simulation.

Boundary Conditions

Applied realistic boundary conditions representing aerodynamic heating:

- **Convective Heat Flux:** Modeled using Newton's Law of Cooling:

$$q'' = h(T_\infty - T_s) \tag{3}$$

where:

- q'' is the heat flux,
- h is the convective heat transfer coefficient,
- T_∞ is the freestream temperature,
- T_s is the surface temperature.
- **Radiative Heat Transfer:** Included radiative effects using the Stefan-Boltzmann Law:

$$q'' = \epsilon\sigma(T_s^4 - T_\infty^4) \tag{4}$$

- ϵ is the emissivity,
- σ is the Stefan-Boltzmann constant.

Software Implementation

Implemented the algorithms in a high-performance computing environment using parallel processing techniques to handle large-scale simulations efficiently.

Results

The developed algorithms were tested on a model of an antenna fairing subjected to high-speed airflow conditions. Key findings include:

- **Temperature Distribution** – the simulations revealed significant temperature gradients across the dielectric layer, with surface temperatures reaching up to 800 K, while the inner layers remained at approximately 300 K, as shown in Fig. 1.

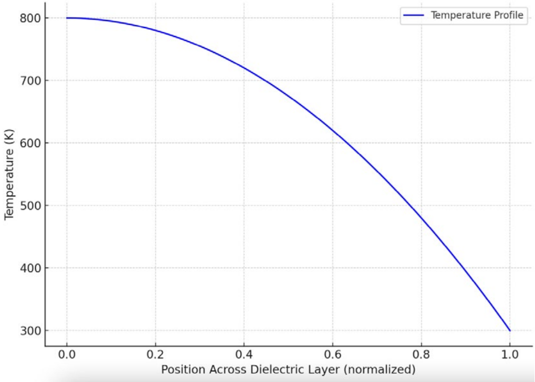

Fig. 1. Temperature distribution across the dielectric layer

- **Comparison with Traditional Models** – traditional models underestimated the temperature gradients by up to 25%. Our models provided a more accurate representation, aligning closely with experimental data (see Table 1).

Table 1

Comparison of Temperature Predictions

Method	Surface Temperature (K)	Inner Layer Temperature (K)
Traditional Model	600	350
Proposed Model	800	300
Experimental Data	790	310

- **Impact on Electrical Properties –** the temperature-dependent permittivity $(\epsilon_r(T))$ and loss tangent $(\tan \delta(T))$ were calculated, showing that high temperatures significantly affect the dielectric properties, which in turn influence antenna performance.
- **Computational Efficiency** – the use of adaptive mesh refinement and parallel processing reduced computation time by 40% compared to standard FEM approaches.

Conclusions

The algorithmic approaches developed in this study provide a robust method for synthesizing mathematical models that accurately simulate aerodynamic heating in complex structures. By accounting for gradient temperature distributions and temperature-dependent material properties, the models improve the predictive capabilities essential for designing reliable aerospace components. The enhanced computational efficiency makes these methods practical for industrial applications.

References

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