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Multi-criteria assessment of aircraft control quality

The proposed is a multi-criteria evaluation of aircraft control quality. The evaluation function includes a scalar convolution of criteria reflecting the decision maker's utility function. A nonlinear compromise scheme and a method for normalizing criteria are presented for both quantitative and qualitative assessment of performance quality.

To assess the quality of functioning of complex objects, the evaluation function $Y[y(x)]$ is used, where $y(x) = \{y_k(x)\}_{k=1}^{s}$ is the vector of quality criteria;$x = \{x_i\}_{i=1}^{n}$ is the vector of independent variables. In the analysis problem, the evaluation function quantitatively expresses the quality measure of a multi-criteria object for given values of arguments x. This is a scalar convolution of criteria, reflecting the utility function of the decision maker (DM).

In [1], a scalar convolution constructed using a nonlinear compromise scheme is proposed:

$$Y(\alpha, y) = \sum_{k=1}^{s} \alpha_k [A_k - y_k(x)]^{-1}; \ \alpha \in \Gamma_\alpha, \qquad (1)$$

where A_k are the constraints on the criteria; $\alpha_k = const$ are the formal parameters defined on the simplex and having a dual physical meaning. On the one hand, these are the weight coefficients expressing the decision maker's preferences for individual criteria. On the other hand, these are the coefficients of the meaningful regression model of the decision maker's utility function, constructed using the concept of a nonlinear compromise scheme.

If the vector of criteria $y(x)$ is normalized by the vector of constraints A,

$y_0(x) = \{y_k(x)/A_k\}_{k=1}^{s} = y_{0k}(x)_{k=1}^{s}$,

then scalar convolution $Y[y_0(x)]$ is applied.

Unlike optimization problems, multi-criteria evaluation belongs to the class of *analysis* problems. Here, convolution (1) is not a target function, but an evaluation function, and its value quantitatively expresses the quality measure of a multi-criteria object for given values of arguments x.

In multi-criteria evaluation of alternatives, it is often necessary to obtain not only an analytical but also a *qualitative* assessment. To do this, the scalar convolution expression $Y(\alpha, y_0)$ should be normalized and the resulting value Y_0 should be correlated with the gradations of the inverted normalized fundamental scale. The general concept of an ordinal fundamental scale is described in [2]. The interval normalized inverted scale is presented in Table 1. It shows the relationship between the qualitative gradations of the properties of objects and the corresponding quantitative assessments y_0 and Y_0.

Table 1

The interval normalized inverted scale

Quality category	**Intervals of the inverted normalized fundamental rating scale y_0 and Y_0**
Unacceptable	1,0 – 0,7

Low	0,7 – 0,5
Satisfactory	0,5 – 0,4
Good	0,4 – 0,2
High	0,2 – 0,0

The design of the nonlinear compromise scheme allows us to normalize the scalar convolution not to the maximum (usually unknown), but to the *minimum* value. Putting in the expression for the nonlinear scalar convolution (1) the ideal (zero) values of the minimized criteria $y_{0k}(x) = 0$ and taking into account the normalization on the simplex of the weight coefficients $\sum_{k=1}^{s} \alpha_k = 1$, we obtain $Y_{0min} = 1$. The formula for the normalized minimized scalar convolution has the form

$$Y_0 = 1 - \frac{1}{Y(\alpha, y_0)} \qquad (2)$$

A qualitative (linguistic) assessment of an alternative is obtained by comparing the analytical assessment of Y_0 with the inverted normalized fundamental scale. Assessment of options on a single normalized fundamental scale makes it possible to solve multi-criteria problems of both traditional formulations and in the case when it is necessary to select an alternative from a set of heterogeneous alternatives for which it is impossible to formulate a single set of quantitative assessment criteria, as well as for assessing a single (unique) alternative.

We will demonstrate the capabilities of a nonlinear compromise scheme in a multi-criteria analysis problem, namely in the problem of assessing the quality of the glide descent process according to several criteria during *aircraft landing.*

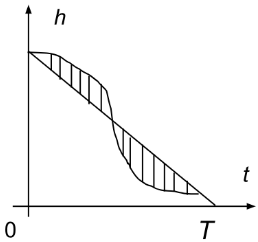

Fig. 1. The change in the altitude h of the aircraft during the glide slope in the coordinates (*h,t*)

Fig. 1 schematically shows the change in the altitude h of the aircraft during the glide slope in the coordinates (h, t). It is assumed that at the moment of time $t=T$ the altitude h is zero. The change in the position of the aircraft b relative to the runway centerline in the lateral plane during the glide slope can be similarly shown.

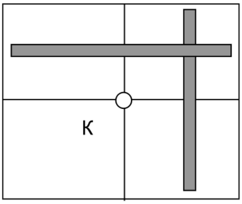

Fig. 2. Diagram of the director control indicator for flight control during landing

When the aircraft descends along the glide path, the pilot controls the aircraft using the director instrument, schematically shown in Fig. 2. The position of the bars shown in the figure means that the aircraft is above the glide path and to the right of the runway centerline. Control consists of aligning the crosshairs of the bars with the central point of the instrument K.

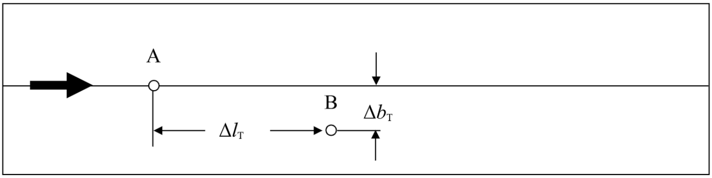

Fig. 3. Diagram of the aircraft's touchdown point on the runway and deviation from the calculated trajectory

Fig. 3. shows that at time $t = T$ the aircraft touched the runway at point B, located at a distance Δl_T from the calculated point A and at a distance Δb_T from the runway centerline.

To assess the quality of the aircraft landing, we will use three terminal (*t*=*T*) quality criteria ($y_1 - y_3$):

$y_1 = |\Delta l_T| < A_1$ – deviation modulus from the calculated point of contact in the longitudinal plane;

$y_2 = |\Delta b_T| < A_2$ – deviation modulus of the point of contact from the longitudinal axis of the runway in the lateral plane;

$y_3 = V_h^{(T)} < A_3$ – vertical speed at the terminal point,

and two integral criteria ($y_4 - y_5$):

$y_4 = \frac{1}{T}\int_0^T |\Delta h| dt < A_4$ – average deviation from the glide path in the vertical plane;

$y_5 = \frac{1}{T}\int_0^T |\Delta b| dt < A_5$ – average deviation from the glide path in the horizontal plane.

In addition, the quality of the aircraft landing process can be characterized by the following criteria: y_6 – deviation from the estimated landing speed at the terminal point; y_7 – course angle at the terminal point; y_8 – bank angle at the terminal point; y_9 – deviation from the estimated pitch angle at the terminal point; y_{10} – average rudder consumption on the glide path (integral criterion), etc. We will assume that the last criteria are satisfied for all aircraft landings and are not included here.

To calculate the integral criteria, we will use the method of approximate integration, which is illustrated by the example of criterion y_4. The time interval of descent along the glide path [0,*T*] is divided into N subintervals Δt, during each of which the value $|\Delta h|_i, i \in [1, N]$ is measured and assumed to be constant. Then in the formula for the integral criterion, we can move from the integral to summation

$y_4 = \frac{1}{T}\int_0^T |\Delta h| dt \approx \frac{1}{T}\sum_{i=1}^{N} |\Delta h_i| \Delta t_1$

If all subintervals are the same, i.e. $\forall i \Delta t_i = \Delta t$, then $T = N\Delta t$ and

$y_4 \approx \frac{\Delta t}{N\Delta t}\sum_{i=1}^{N}|\Delta h_i| = \frac{1}{N}\sum_{i=1}^{N} |\Delta h_i|.$

The criterion y_5 is calculated in a similar way.

As an evaluation function we use the scalar convolution of criteria (1). The weight coefficients α can be determined in the interactive procedure described in [3]. Let the following values of the weight coefficients be obtained:

$\alpha_1 = 0{,}25;\ \alpha_2 = 0{,}22;\ \alpha_3 = 0{,}28;\ \alpha_4 = 0{,}16;\ \alpha_5 = 0{,}09.$

The following values of the criteria restrictions are specified (recall that this is a model example):

$A_1 = 15m;\ A_2 = 10m;\ A_3 = 1m\ /\ sec;\ \ A_4 = 30m;\ A_1 = 20m.$

Next, using the nonlinear compromise scheme method, we will evaluate the quality of two aircraft landings with different numerical values of partial criteria.

Landing 1: Let $y_1 = 6m;\ y_2 = 3m;\ y_3 = \frac{0{,}2m}{sec};\ y_4 = 10{,}5m; y_5 = 7{,}25m.$

Normalization according to the formula $y_{0k} = \frac{y_k}{A_k}, k \in [1, s]$ gives the values of relative partial criteria:

$y_{01} = 0{,}4;\ y_{02} = 0{,}3;\ y_{03} = 0{,}2;\ y_{04} = 0{,}35;\ y_{05} = 0{,}36;$

Let's calculate the scalar convolution of criteria using a nonlinear compromise scheme

$$Y = 0{,}25\frac{1}{1-0{,}4} + 0{,}22\frac{1}{1-0{,}3} + 0{,}28\frac{1}{1-0{,}2} + 0{,}16\frac{1}{1-0{,}35} + 0{,}09\frac{1}{1-0{,}36} = 1{,}47$$

Taking into account the normalization according to formula (2), we have

$$Y_0 = 1 - \frac{1}{1{,}47} = 0{,}32$$

Comparison of this value with the qualitative gradations of the inverted normalized fundamental scale (Table 1) allows us to conclude that this fit can be assessed as *good.*

Landing 2: Let $y_1 = 3m;\ y_2 = 4m;\ y_3 = 0{,}6m;\ y_4 = 13{,}25m; y_5 = 10{,}5m.$ Relative partial criteria: $y_{01} = 0{,}2;\ y_{02} = 0{,}4;\ y_{03} = 0{,}6;\ y_{04} = 0{,}44;\ y_{05} = 0{,}52.$

The nonlinear compromise scheme gives

$$Y = 0{,}25\frac{1}{1-0{,}2} + 0{,}22\frac{1}{1-0{,}4} + 0{,}28\frac{1}{1-0{,}6} + 0{,}16\frac{1}{1-0{,}44} + 0{,}09\frac{1}{1-0{,}52} = 1{,}85$$

Normalization according to formula (4) gives

$$Y_0 = 1 - \frac{1}{1{,}85} = 0{,}46$$

According to the inverted normalized fundamental scale, landing 2 is assessed as *satisfactory*. This conclusion follows from the criterion y_3=0.6 m/sec (hard landing).

The described procedure of multi-criteria assessment is applicable, in particular, to the training and education of pilots and in similar cases in other subject areas.

References

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