UDC 331.101.1:629.735.083(076.5):656.7.086(045)

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Air transport technologies resources optimal recombination through variational and subjective analysis approach

Application of the calculus of variations simplest problem setting allows finding the optimal path between two points with respect to the minimal time of the air transportation as a significant resource of the air transportation technologies. The proposed recombination of the resources is based upon the optimal distribution of the individuals' subjective preferences obtained with the use of the subjective analysis theory key point referred to a conditional optimality of the preferences entropy.

Introduction

The time of transportation is an important resource of the air transport technologies. Factually, it is the matter of the individuals' subjective preferences [1] whether to take one or another transport technology. The principle of the subjective entropy conditional optimality mathematically adopted (derived) from the Jaynes' principle [2]–[4], which is a well known in theoretical physics. However, in subjective analysis [1], the mentioned above mathematical formalism of [2]–[4] is implemented into the realm of social sciences such as economics, psychology, sociology, politics etc. The approach of the individuals' preferences conditional optimality from the subjective analysis [1] has its outcomes in neural networks [5].

Herewith, it is proposed to discuss a type of a combined problem setting in regards with the variational objectivity and available alternatives preferences functions subjectivity. That implies some applications likewise in references [6]–[11].

Development of the approach

In the framework of the simplest problem of calculus of variations, the time (one of the air transport technologies resources) can be expressed as

$$dt = \frac{ds}{v(x)}, \qquad (1)$$

where dt is the differential of time t; ds is the differential of path; $v(x)$ is the instantaneous speed as a function of the coordinate x of the corresponding reference system.

Surely, the differential equation (1) follows the differential equation of a material particle plane motion in the natural form of the motion description:

$$v = \frac{ds}{dt}. \qquad (2)$$

Then, transforming from (1) and (2)

$$dt = \frac{\sqrt{dx^2 + dy^2}}{v_0 + kx}, \qquad (3)$$

where dx and dy are the differentials of the x and y coordinates correspondingly; v_0 is the initial speed; k is the coefficient of proportionality (the rate of the speed varying (change) with respect to the x coordinate).

Integrating (3)

$$T[y]=\int_0^{d_1}\frac{\sqrt{1+(y'_x)^2}}{v_0+kx}dx\,, \qquad (4)$$

where $T[y]$ is the objective functional (the time of the air transportation, the indispensable element of the air transport technology, important air transport technology resource) dependent upon the unknown (free, to be found/determined) function of $y(x)$; d_1 is the terminal value of x varying;

$$y'_x=\frac{dy}{dx}\,. \qquad (5)$$

Solving (4) for $y(x)$, with the use of the Euler-Lagrange equation:

$$\frac{\partial F}{\partial y}-\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)=0\,, \qquad (6)$$

where

$$F=\frac{\sqrt{1+(y'_x)^2}}{v_0+kx}\,; \qquad (7)$$

$$y'\equiv y'_x\,, \qquad (8)$$

one can obtain

$$y(x)=C_1\pm\sqrt{\frac{1}{(kC)^2}-\left(\frac{v_0}{k}+x\right)^2}\,, \qquad (9)$$

where

$$C_1=\sqrt{\frac{k^2\left[(y_1)^2+2\frac{v_0}{k}d_1+(d_1)^2\right]^2+(2v_0y_1)^2}{(2ky_1)^2}-\left(\frac{v_0}{k}\right)^2}\,, \qquad (10)$$

where

$$y_1=y(d_1)\,; \qquad (11)$$

$$C=\frac{2y_1}{\sqrt{k^2\left[(y_1)^2+2\frac{v_0}{k}d_1+(d_1)^2\right]^2+(2v_0y_1)^2}}\,. \qquad (12)$$

Numerical example is illustrated in Fig. 1.

The magnitudes of the values for the quantitative experimentations by (1)-(12) are conditional.

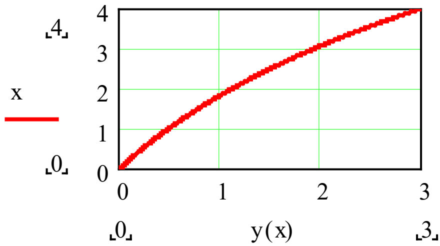

Fig. 1. Extremal of $y(x)$

The preferences distribution is shown in Fig. 2.

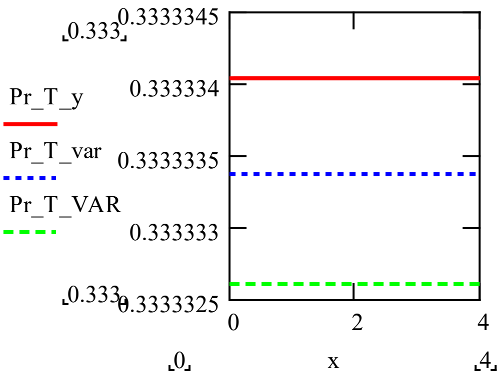

Fig. 2. Preferences of the alternatives

In Fig. 2 it is depicted Pr_T_y for the preference function with the extremal solution of (9) with (10)-(12) used in (4).

Pr_T_var stands for a "smaller" variation of the extremal solution of (9).

Pr_T_VAR is a "greater" variation of (9).

As to the individuals' subjective preferences distribution, shown in Fig. 2, it is worth saying that for the simulated alternatives the magnitudes significantly depend upon the cognitive parameter β, [1]:

$$\pi_{(i)} = \frac{e^{-\beta T\left[y_{(i)}\right]}}{\sum_{j=1}^{3} e^{-\beta T\left[y_{(j)}\right]}}, \qquad (13)$$

where $\pi_{(i)}$ is the preference function of the i^{th} alternative; $T\left[y_{(i)}\right]$ is the time for the corresponding alternative (variated $y(x)$ function).

The shorter time has the higher preference since

$$\beta > 0\,. \tag{14}$$

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