UDC 629.7.05:531.1

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Technologies for increasing the accuracy of SINS algorithms: optimization of the 4th order Panov algorithm

The Panov algorithm of the 4th order of accuracy was studied. New parameter values for this algorithm are derived that are adaptable for different motion types. By implementing the optimized algorithm on the four-frequency quaternion model, navigation information was obtained with greater accuracy than with the classical parameter value.

Adaptation of existing algorithms for objects with complex types of movement

Strapdown inertial navigation systems (SINS) allow accurate determination of the position, speed and orientation of moving objects without the use of external sources of information. They have proven their effectiveness in conditions where traditional navigation methods can fail, for example, due to obstacles or interception of signals.

The main elements of SINS include measuring equipment for collecting data on object movement and a computer (calculator) for their processing. Ideal data from the triad of angular velocity sensors are sent to the on-board computer in the form of quasi-coordinates [1]:

$$\theta_n^* = \int_{t_{n-1}}^{t_n} \boldsymbol{\omega}(t)dt, \qquad (1)$$

$\boldsymbol{\omega}(t)$ – projections of the absolute angular velocity vector of the object on the axis of the bound coordinate system.

Based on the measurements, an orientation determination algorithm is implemented that calculates the position of the object in real time. The main quality criterion of SINS is the accuracy of the navigation parameters obtained using the selected algorithm.

There are many algorithms for determining orientation quaternions and they differ in accuracy, mathematical complexity, and approaches to the use of intermediate parameters, such as the orientation vector or the Euler vector.

In Ukraine, a significant contribution to the development of algorithms for determining the orientation vector was made by A. Panov [2-4], who created a number of algorithms that use primary information (1) from different recording cycles. This approach allows you to take into account the features of the object's movement, but requires data from several previous cycles, which can load the computer at the beginning of the system's operation.

Foreign research is focused on algorithms that use inertial information within a calculation cycle, in particular, polynomial approximation of angular velocity, which is especially useful for highly dynamic objects [5-8].

Most algorithms for determining the orientation vector are adapted to the motion of a rigid body corresponding to regular precession or conical rotation.

Since the local or accumulated error of the algorithm has an analytical form, it allows to optimize existing orientation algorithms for specific test movements (different from classical ones).

One of the methods of optimization of algorithms for regular precession and conical motion is proposed by A. Panov [4] and focuses on minimizing the asymptotic estimates of the error of the computational drift. The improved approach of the optimization technique is described in works [6-8], where the results of the adaptation of algorithms, including for random angular motion, are presented.

R. Miller [5] proposed optimization of orientation algorithms for conical motion by adjusting the coefficients without changing the structure of the algorithm, using an analytical expression for the error in the form of a power series. A similar technique is presented in the work of M. Ignagni [6].

Choosing the optimal algorithm for a specific moving object is a complex task that requires detailed analysis. The accuracy of the algorithms is usually evaluated through special test motions of a rigid body, where the orientation and angular velocity components are presented analytically. Of particular interest is obtaining methodical accuracy estimates of algorithms optimized for more complex rotational movements.

Optimization of the Panov algorithm

The choice of the algorithm for determining orientation quaternions in BINS depends on the criteria put forward for the navigation system. At the initial stage, the parameters that describe the position of the object relative to its center of mass are analyzed, and the features of the system are also taken into account, such as the type of orientation parameters, the characteristics of the sensors, the speed of the algorithm and the level of accuracy. In the development of modern SINS, algorithms of the 4th order are usually used, which have high accuracy and at the same time do not accumulate additional noise. But they may have errors due to the chosen method of numerical integration.

One of the widely used algorithms for determining orientation in SINS is the algorithm of A. Panov of the 4th order of accuracy, which in general has the following form [2]:

$$\theta_{\mathrm{i},n} = \theta^{*}_{\mathrm{i},n} + \left(\frac{2}{3} + \alpha\right)\left(\theta_{\mathrm{i}-\frac{1}{2}\Delta t,n} \times \left(\theta^{*}_{\mathrm{i},3} - \theta_{\mathrm{i}-\frac{1}{2}\Delta t,n}\right)\right), \quad (3)$$

де θ^{*}_{i}- integrals from the angular velocity projections obtained at each step of information acquisition:

$\theta^{*}_{\mathrm{i},n} = (\theta^{*}_{\mathrm{i},1}, \theta^{*}_{\mathrm{i},2}, \theta^{*}_{\mathrm{i},3})$

$\theta_{\mathrm{i}-\frac{1}{2}\Delta t}$- quasi-coordinate values obtained within the selected cycle;

α – constant parameter of the algorithm.

In order to increase the accuracy of the Panov algorithm and optimize it for the conditions of use on objects with complex types of angular motion, a study was conducted that focused on the special setting of the α parameter for specific implementation conditions.

Program-numerical optimization on the four-frequency model

When developing and testing algorithms for determining orientation, reference models based on the trigonometric representation of the quaternion are used. In this work, a new continuous reference model based on the four-frequency representation of the orientation quaternion is presented as a test motion [9]:

$$\begin{aligned}
\lambda_0(t) = {} & cos\frac{\chi}{2}\left(cos\frac{\varphi}{2}cos\frac{\psi}{2}cos\frac{\vartheta}{2} + sin\frac{\varphi}{2}sin\frac{\psi}{2}sin\frac{\vartheta}{2}\right) \\
& - sin\frac{\chi}{2}\left(cos\frac{\varphi}{2}sin\frac{\psi}{2}cos\frac{\vartheta}{2} + sin\frac{\varphi}{2}cos\frac{\psi}{2}sin\frac{\vartheta}{2}\right); \\
\lambda_1(t) = {} & cos\frac{\chi}{2}\left(cos\frac{\varphi}{2}cos\frac{\psi}{2}sin\frac{\vartheta}{2} - sin\frac{\varphi}{2}sin\frac{\psi}{2}cos\frac{\vartheta}{2}\right) \\
& - sin\frac{\chi}{2}\left(sin\frac{\varphi}{2}cos\frac{\psi}{2}cos\frac{\vartheta}{2} - cos\frac{\varphi}{2}sin\frac{\psi}{2}sin\frac{\vartheta}{2}\right); \\
\lambda_2(t) = {} & cos\frac{\chi}{2}\left(cos\frac{\varphi}{2}sin\frac{\psi}{2}cos\frac{\vartheta}{2} + sin\frac{\varphi}{2}cos\frac{\psi}{2}sin\frac{\vartheta}{2}\right) \\
& + sin\frac{\chi}{2}\left(cos\frac{\varphi}{2}cos\frac{\psi}{2}cos\frac{\vartheta}{2} + sin\frac{\varphi}{2}sin\frac{\psi}{2}sin\frac{\vartheta}{2}\right); \\
\lambda_3(t) = {} & cos\frac{\chi}{2}\left(sin\frac{\varphi}{2}cos\frac{\psi}{2}cos\frac{\vartheta}{2} - cos\frac{\varphi}{2}sin\frac{\psi}{2}sin\frac{\vartheta}{2}\right) \\
& + sin\frac{\chi}{2}\left(cos\frac{\varphi}{2}cos\frac{\psi}{2}sin\frac{\vartheta}{2} - sin\frac{\varphi}{2}sin\frac{\psi}{2}cos\frac{\vartheta}{2}\right),
\end{aligned} \tag{4}$$

де φ, ψ, ϑ and χ - angles that are dependent on time:
$\varphi = k_1 t, \psi = k_2 t, \vartheta = k_3 t, \chi = k_4 t.$

The obtained quaternion representation of the rotational motion is supplemented with expressions for the projections of the angular velocity vector, which are calculated from the inverse quaternion kinematic equation (5):

$$\omega = 2 \cdot \tilde{\Lambda} \circ \dot{\Lambda}, \tag{5}$$

де $\tilde{\Lambda}$ – conjugate quaternion inverse of the given:
$\tilde{\Lambda} = (\lambda_0, -\lambda_1, -\lambda_2, -\lambda_3)$;
$\dot{\Lambda} = \frac{d\Lambda}{dt}$;
$\omega = (0, \omega_1, \omega_2, \omega_3)$.

By integrating expressions (5) at each cycle $[t_{n-1}, t_n]$ and with a properly specified set of frequencies k_i, a reference model is obtained that can be used to simulate the angular motion of a rigid body under various operating conditions

To assess the accuracy of the selected algorithm, SINS usually determines the drift type error (accumulated small angle of rotation of the calculated position of the object relative to the true one). This error is permanent and accumulates over time [1]. Drift analysis for a specific moving object is important for managing and correcting its position.

In this work, different values of the parameter α were determined for Panov's algorithm. The study was conducted on a four-frequency model in the case when the k_i parameters are selected taking into account the limitation for the angular velocity module at the study interval (system operation time T=1000 seconds, information acquisition cycle Δt=0.1 seconds). The results of the numerical implementation of the algorithm on the four-frequency model are presented in Table 1.

Table 1 – The value of the drift error at different parameter values α

parameter α	Panov's algorithm of the 4th order (drift, rad)
0	0.0006341
0.001	0.0000271
0.002	0.0005095
0.0015	0.0002423
-0.001	0.0010945
0.0005	0.0002927

The results show that under the same conditions of implementation on the four-frequency model, the smallest drift is obtained at the value α=0.001. In addition, the performed optimization made it possible to increase the accuracy of the algorithm in comparison with the classical values for the Miller algorithm [5-6, 9].

Graphically, the relationship between the value of the parameter α and the obtained drift error is shown in Fig. 1

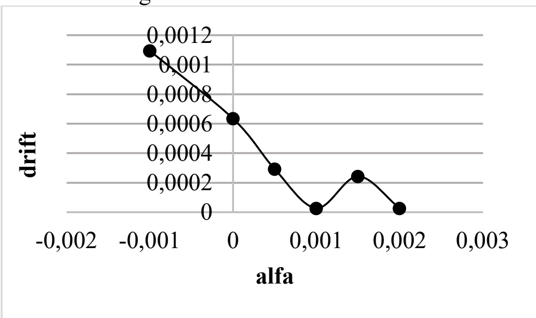

Figure 1 – Accumulated drift estimate for the modified Panov algorithm depending on the parameterα

Conclusions

The Panov algorithm of the 4th order of accuracy was studied. With the help of the technique of special setting of parameters, this algorithm was adapted to the conditions of motion, which are more complex than classical ones (regular precession and conical rotation). The optimal value of the alpha parameter is established, which allows to increase the accuracy of determining the navigation parameters in systems with four degrees of freedom, compared to the classical approaches of Panov's algorithm and Miller's algorithm.

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