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Optimization of algorithms for determining orientation quaternions on the reference model of spherical motion of a rigid body in Krylov angles

The results of the optimization of three well-known algorithms of orientation on the analytical model of the spherical motion of a rigid body, which is based on a sequence of Krylov angles that change over time according to a linear law, are presented. It is shown that the optimized algorithms have improved estimates of the accumulated computed drift compared to the non-optimized algorithms.

The problem of improvement of accuracy of the algorithms for determining the orientation quaternion under conditions of ideal information from the triad of angular velocity sensors in the form of quasi-coordinates is considered:

$$\theta_i^* = \int_{t_{n-1}}^{t_n} \omega_i(t) dt, \quad i = 1, 2, 3, \quad (1)$$

where $\omega_i(t)$, $i = 1, 2, 3$ – projections of the absolute angular velocity vector $\vec{\omega}(t)$ on the axis of the coordinate system associated with the moving object.

The use of quaternions in strapdown orientation systems is prioritized, since the kinematic equation is linear, has the first integral, and quaternion algebra can be used for vector transformations. One of the approaches to the construction of algorithms for calculating the orientation quaternion is based on the use of the orientation vector as an "intermediate" parameter [1, 2]. The conical movement model is usually used to evaluate the coefficients in order to optimize the algorithms for determining the increment of the orientation vector per calculation cycle. Algorithms optimized for conical motion are not optimal for other rotational motions. This work presents the results of optimization of three well-known orientation algorithms on the model of spherical motion of a rigid body, which is built on a sequence of Krylov angles.

An analytical model of the spherical motion of a rigid body based on the sequence of Krylov angles.

Consider a kinematic model built on the basis of a sequence of elementary rotations around the coordinate axes in terms of Krylov angles, which change over time according to a linear law [3]:

$$\lambda_0(t) = c_1 \cdot c_2 \cdot c_3 + s_1 \cdot s_2 \cdot s_3; \quad \lambda_1(t) = c_1 \cdot c_2 \cdot s_3 - s_1 \cdot s_2 \cdot c_3;$$

$$\lambda_2(t) = c_1 \cdot s_2 \cdot c_3 + s_1 \cdot c_2 \cdot s_3; \quad \lambda_3(t) = s_1 \cdot c_2 \cdot c_3 - c_1 \cdot s_2 \cdot s_3, \quad (2)$$

where $c_i = \cos\left(\frac{k_i t}{2}\right)$, $s_i = \sin\left(\frac{k_i t}{2}\right)$, $i = 1, 2, 3$, $\lambda_j(t)$, $j = 0, 1, 2, 3$ – components of the orientation quaternion, $k_i > 0$, $i = 1, 2, 3$ – constant values that can be interpreted as frequencies.

According to the kinematic equation, the quaternion (2) corresponds to the angular velocity vector with the following formulas for projections onto the connected axes:

$$\begin{aligned} \omega_1(t) &= k_3 - k_1 \cdot \sin(k_2 t); \\ \omega_2(t) &= k_1 \cdot \cos(k_2 t) \cdot \sin(k_3 t) + k_2 \cdot \cos(k_3 t); \\ \omega_3(t) &= k_1 \cdot \cos(k_2 t) \cdot \cos(k_3 t) - k_2 \cdot \sin(k_3 t). \end{aligned} \quad (3)$$

Formulas (2), (3) can be interpreted as exact solutions of the system of dynamic and kinematic equations of a rigid body rotational motion. At the same time, the rotation model is formal, since it is not possible to specify in advance the dynamic characteristics of the rigid body for which it takes place. It is shown in [3] that model (2), (3) with given values of frequencies k_1, k_2, k_3 describes a more complex rotational motion than the known formal model of conical motion. For the modeling of orientation algorithms, formulas (3) are used as the basis of the analytical representation of quasi-coordinates at the calculation cycle.

Fig.1 shows the trajectories $\lambda_i(\lambda_0), i = 1,2,3$ in the configuration space of the orientation parameters obtained numerically for the kinematic model (2) on the time interval $t \in [0,200]$ sec and at the frequency values $k_1 = 0.15, k_2 = 0.25, k_3 = 0.05$.

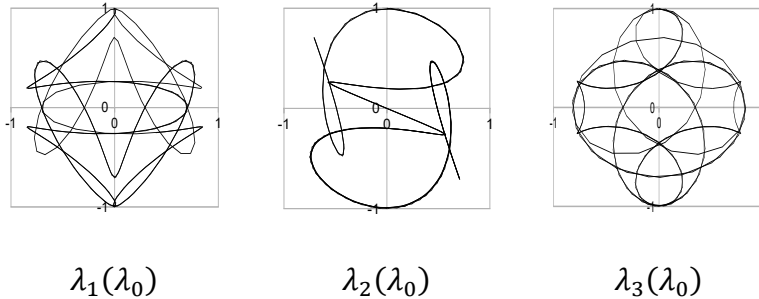


Fig. 1. Trajectories in the configuration space for the spherical motion model

Selected orientation algorithms for optimization on spherical motion model.

Consider the following algorithms for optimization on the model of spherical motion.

1. Algorithm 1 (R. Miller [1]).

In Miller's algorithm, the increment of the orientation vector $\vec{\theta}_n = (\theta_{n1}, \theta_{n2}, \theta_{n3})$ per computational cycle $[t_{n-1}, t_n]$ is calculated from the formula:

$$\vec{\theta}_n = \vec{\theta}_n^* + \alpha(\vec{\theta}_n^{(1)} \times \vec{\theta}_n^{(3)}) + \beta\vec{\theta}_n^{(2)} \times (\vec{\theta}_n^{(3)} - \vec{\theta}_n^{(1)}), \quad (4)$$

where $\vec{\theta}_n^{(1)} = \int_{t_{n-1}}^{t_{n-1}+1/3\Delta T} \vec{\omega}(t)dt$, $\vec{\theta}_n^{(2)} = \int_{t_{n-1}+1/3\Delta T}^{t_{n-1}+2/3\Delta T} \vec{\omega}(t)dt$, $\vec{\theta}_n^{(3)} = \int_{t_{n-1}+2/3\Delta T}^{t_{n-1}+\Delta T} \vec{\omega}(t)dt$ - output signals of gyroscopes formed within the calculation cycle at moments of time $t_{n-1} + 1/3\Delta T$, $t_{n-1} + 2/3\Delta T$, $t_{n-1} + \Delta T$, where ΔT - calculation cycle time, $\vec{\theta}_n^* = (\theta_{n1}^*, \theta_{n2}^*, \theta_{n3}^*)$. Miller obtained that $\alpha = 33/80$, $\beta = 57/80$, $\alpha + \beta = 90/80$ on the basis of optimization for conical motion.

2. Algorithm 2 (A. Panov [2]).

In Panov's two-step algorithm, the increment of the orientation vector per calculation cycle is calculated from the formula:

$$\vec{\theta}_n = \vec{\theta}_n^* + \alpha(\vec{\theta}_n^{(4)} \times \vec{\theta}_n^*), \quad (5)$$

where $\vec{\theta}_n^{(4)} = \int_{t_{n-1}}^{t_{n-1}+1/2\Delta T} \vec{\omega}(t)dt$, $\alpha = 2/3$.

To obtain the current orientation quaternion Λ_n^* based on Algorithm 1 and Algorithm 2, we apply the formula for adding turns $\Lambda_n^* = \Lambda_{n-1}^* \circ \Delta\Lambda_n^*$, where the scalar $\Delta\lambda_{n0}^*$ and vector parts $\overrightarrow{\Delta\lambda}_n^*$ of the turn quaternion $\Delta\Lambda_n^*$ on the cycle $[t_{n-1}, t_n]$ are calculated according to the formulas

$$\begin{aligned} \Delta\lambda_{n0}^* &= 1 - (1/8)\theta_n^2 + (1/384)\theta_n^4, \\ \overrightarrow{\Delta\lambda}_n^* &= (1/2)\overrightarrow{\theta}_n(1 - \theta_n^2/24), \end{aligned} \quad (6)$$

where $\theta_n^2 = \theta_{n1}^2 + \theta_{n2}^2 + \theta_{n3}^2$.

3. Algorithm 3 (Yu. Plaksyiy [4]).

This two-step power-law algorithm for calculating the rotation quaternion does not use intermediate parameters and has working formulas for determining the components of the rotation quaternion directly from the quasi-coordinate information in the form:

$$\Delta\lambda_{n0}^* = 1 - (1/8)\theta_n^{*2} + (1/384)\theta_n^{*4},$$

$$\vec{\Delta\lambda}_n^* = (1/2)\vec{\theta}_n^*(1 - \theta_n^{*2}/24) + \alpha(\vec{\theta}_n^{(4)} \times \vec{\theta}_n^*), \quad (7)$$

where $\alpha = 1/3$.

Numerical optimization of orientation algorithms based on the analytical model of spherical motion

To optimize the algorithms, consider the rotational movement model, which is described by formulas (3), (4) at the values $k_1 = 0.15, k_2 = 0.25, k_3 = 0.05$.

The numerical experiment was carried out on the time interval $t \in [0,200]$ sec. for the calculation cycle $\Delta t = 0.1$ sec. It was experimentally found that the accumulated error of computational drift has a trend of linear growth over time for all algorithms under consideration.

Table 1 presents the maximum values of the accumulated drift error for the presented algorithms (5), (6), (7) on the time interval $t \in [0,200]$ sec. for the calculation cycle $\Delta t = 0.1$ sec.

Table 1.

The maximum error value of the accumulated drift, rad

Algorithm 1, $\alpha = 33/80, \beta = 57/80$	Algorithm 2, $\alpha = 2/3$	Algorithm 3, $\alpha = 1/3$
1.073E-06	1.029E-06	0.78E-06

Optimization of the algorithms based on the minimization of the error of the accumulated computational drift was carried out by refining the coefficients in the algorithms. It was found that the optimized coefficients differ significantly from the initial ones for Algorithm 1. The specified values of the coefficients for Algorithm 1 are as follows: $\alpha = -151/20, \beta = 347/40$. Optimization for Algorithm 2 gives the value of the coefficient $\alpha = 4000321/6000000$, for Algorithm 3 we have as a result a refined coefficient $\alpha = 99994/300000$.

The values of the refined coefficients for the algorithms under consideration are given in Table. 2.

The performed numerical experiment shows that the accumulated error of the computational drift for all optimized algorithms has an oscillatory character, in contrast to the respective error for non-optimized algorithms. At the same time, the maximum value of the error practically does not increase over time. Table 2 presents the maximum error values of the accumulated drift for the optimized algorithms on the time interval $t \in [0,200]$ sec.

Table 2.

The maximum value of the accumulated drift error for optimized algorithms, rad		
Algorithm 1, $\alpha = -151/20$, $\beta = 347/40$	Algorithm 2, $\alpha = 4000321/6000000$	Algorithm 3, $\alpha = 99994/300000$
0.225E-06	0.421E-07	0.471E-07

Conclusions.

On the basis of the numerical experiment performed on an analytical model of spherical rotational motion in terms of linear Krylov angles, refined coefficients were found for the selected orientation algorithms that optimize the error of the maximum accumulated computational drift. The performed optimization leads to a reduction by an order of magnitude of the maximum value of the accumulated error of the computational drift and to an improvement of the change tendency of the computational drift error over time.

References

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