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Determination of the longitudinal moment coefficient of an aircraft with wing tips

Using the expressions for the coefficient of the longitudinal moment for an aircraft with a wing without wingtips, the longitudinal moment coefficient for the aircraft with a wing with aerodynamic ends was recorded, and its derivatives were calculated.

The lifting force of an aircraft wing is generated by the pressure difference between the lower and upper surfaces. Air moves from the high-pressure area beneath the wing to the low-pressure area above, creating vortices at the wingtips. These vortices redistribute lift along the wing, reducing effective area and increasing drag. These currents, forming with the main – in front flow, form powerful vortex bundles of air, pulling the aircraft back from the ends of the wing. End vortices lead to a redistribution of the lifting force along the span of the wing, reducing its effective area and elongation, form an inductive resistance and reduce the aerodynamic quality.

To reduce these negative factors on the ends of the wing of many modern aircraft, aerodynamic wingtips are installed (end washers or Witecomb washers, AT Winglets, sharklets). It is proved that these designs increase the effective span of the wing, reduce the inductive resistance, and, as a result, increase the lifting force at the end of the wing, increase the aerodynamic quality and lengthening of the wing, almost without changing its swing [1-4]. They also increase the fuel efficiency of aircraft or the range of flight in gliders. Currently, the same types of aircraft can have different versions of the wingtips.

During flight, aircraft are subjected to various external disturbances, such as turbulence, updrafts, downdrafts, and gusty winds. These factors can alter key parameters like overload, angle of attack, and flight speed. To maintain stability, aircraft must have longitudinal static stability, meaning they can automatically return to their predefined flight mode without pilot intervention, as long as flight speed remains relatively unchanged.

To determine the stability characteristics of an aircraft, it is essential to know its mathematical motion model, which arises under the influence of forces and moments that cause both rotational and translational movement. The aircraft's mathematical model, which allows for the investigation of the stability of its perturbed motion, describes the process of the aircraft's response to control surface deflections or external disturbances.

This model is nonlinear, making it challenging to obtain practical results. In several cases, to simplify the calculations, the motion can be considered steady, and the perturbation can be considered small. This approach will be applied in the present work. As the following studies demonstrate, even in steady motion, accounting for the influence of wing tips on aircraft stability is extremely complex, which is a primary reason for the scarcity of publications in this area.

In this article, we determine the aerodynamic forces acting on the aircraft during steady horizontal flight and the longitudinal moment of these forces relative to the transverse axis passing through the center of gravity for a wing equipped with aerodynamic tips such as AT Winglets.

Longitudinal moment wing with aerodynamic tips

We place the origin of the coordinate system at the center of the wing, with the Oz – axis directed along the span to the right, the Oy – axis upwards, and the Ox –axis aligned with the airflow. The longitudinal (pitch) moments are calculated relative to the axis based on all forces acting on the aircraft. In horizontal flight, the airplane is influenced by the following forces:

– the weight force G acts on the center of gravity and is always directed vertically downward towards the center of the earth;

– the aircraft's lift Y acts at the center of pressure and is directed perpendicular to the direction of the undisturbed airflow;

– the drag force of the aircraft Q – is located on the longitudinal axis and is directed in the direction opposite to the movement of the aircraft;

– the tractive force P is generally directed towards the aircraft along the engine axis;

– the total aerodynamic forces are generated by the upper tips.

The force generated by the left upper end is labeled $\vec{R}_l^u$, and the force generated by the right upper end is labeled $\vec{R}_r^u$.

The projections of these forces onto the axis of the chosen coupled coordinate system are given by the formulas:

$$\begin{aligned} \vec{R}_l^u &= \left\{R_l^u \cos\varphi \sin\alpha_z, R_l^u \sin\varphi + R_l^u \cos\varphi \sin\alpha_z\right\}; \\ \vec{R}_r^u &= \left\{R_r^u \cos\varphi \sin\alpha_z, R_r^u \sin\varphi - R_r^u \cos\varphi \sin\alpha_z\right\}; \end{aligned} \quad (1)$$

– the total aerodynamic forces created by the lower tips, denoted, respectively, the left $\vec{R}_l^l$ and right $\vec{R}_r^l$. The projections of these forces on the coordinate axes are expressed by the formulas

$$\begin{aligned} \vec{R}_l^l &= \left\{R_l^l \cos\varphi \sin\alpha_z, R_l^l \sin\varphi + R_l^l \cos\varphi \sin\beta\right\}; \\ \vec{R}_r^l &= \left\{R_r^l \cos\varphi \sin\alpha_z, R_r^l \sin\varphi - R_r^l \cos\varphi \sin\beta\right\}. \end{aligned} \quad (2)$$

The magnitudes of the forces defined by formulas (1) at the upper left and upper right wingtips are equal. As a result, the indices indicating "left" and "right" for these force magnitudes will not be used in the future. This also applies to the forces defined by formulas (2). Therefore, the magnitudes of the aerodynamic forces produced by one upper and one lower wingtips will be represented in the form.

$$\vec{R}_l^u = \vec{R}_r^u = R^u;$$

$$\vec{R}_l^l = \vec{R}_r^l = R^l.$$

The coefficients of forces R^u and R^l denote, respectively, the letters C^u and C^l i.e.

$$R^u = C^u \frac{\rho V_\infty^2}{2} S \;;$$

$$R^l = C^l \frac{\rho V_\infty^2}{2} S \;.$$

The forces generated by the wingtips are relatively small compared to other forces acting on the aircraft. We will assume that variations in these forces with changes in the angle of attack are also minor.

We will assume that the aircraft's center of gravity is located on the axis Ox at a distance x_T from the nose of the aircraft. The distance from the nozzle to the center of pressure will be denoted as x_D, and the distance to the focus point will be denoted by x_F.

If necessary, we will assume that these points are situated on the mean aerodynamic chord (MAR) and are measured from the leading edge of the MAR. The position of the focus is significantly affected by the Mach number: in the transonic range, there is a sharp rearward shift of the focus, which stabilizes with further increases in Mach number.

We will also assume that the vertical projections of the centers of pressure of all parts of the wingtips intersect with the axis Oz. The distance of the centers of pressure of the upper halves of the wingtips from the axis Oz, will be denoted as y^u, while the centers of pressure of the lower halves will be denoted as y^l.

Consequently, the wingtips together will produce a diving moment relative to the axis Oz:

$$M_z^u = -2y^u R^u \cos\varphi \sin\alpha_z \,,$$

and the lower parts of the wingtips will create a tuning moment

$$M_z^l = -2y^l R^l \cos\varphi \sin\beta \,.$$

Considering the aerodynamic forces on the wingtips in terms of their coefficients and dynamic pressure, we can express:

$$\begin{aligned} M_z^u &= -2y^u C^u \frac{\rho V_\infty^2}{2} \cos\varphi \sin\alpha_z; \\ M_z^l &= -2y^l C^l \frac{\rho V_\infty^2}{2} \cos\varphi \sin\beta. \end{aligned} \tag{3}$$

From these formulas, it is clear that if the angles of collapse of the lower wingtips are such that $\phi = \pi/2$, then only the diving moment from the upper wingtips will be present.

Conversely, if $\phi = \pi/2$, then only the trimming moment from the lower wingtips will be present. Additionally, if a particular collapse angle is zero, the corresponding moment reaches its maximum value.

If the angle of collapse of the lower wingtips $\phi = \pi/2$, then the elongation of the effective centering of the aircraft is greatest at a constant twist angle of the lower wingtip β, and if $\phi = \pi/2$, under the same condition, the displacement of the aircraft alignment does not occur. Similarly, if the angle of cambering of the upper wingtip $\phi = 0$, then the extension of the effective focus of the aircraft is greatest with the unchanged twist angle of the upper wingtip α_z, and if $\phi = \pi/2$, under the same condition, the shift of the effective focus of the aircraft does not occur. Obviously, in the presence of only vertical (upper) wing tips, the effective focus of the aircraft is shifted as much as possible, and the alignment does not change.

It is important to note that for wings with wingtips, the moments (3) are associated with the wing's lifting force and always occur when there is a lifting force.

Furthermore, the angles of twist of the aerodynamic wingtips α fnd β are sufficiently small.

Dividing the expression of the moments into an expression $\frac{\rho V_\infty^2}{2} Sb$ and denoting

$$M_z^u = \frac{M_z^u}{\frac{\rho V_\infty^2}{2} Sb}; \quad M_z^l = \frac{M_z^l}{\frac{\rho V_\infty^2}{2} Sb},$$

we obtain the expressions for the coefficients of the moments created by the wingtips

$$\begin{aligned} m_z^u &= -2\bar{y}^u C^u \cos\varphi \sin\alpha_z ; \\ m_z^l &= -2\bar{y}^l C^l \cos\varphi \sin\beta , \end{aligned} \qquad (4)$$

where $\bar{y}^u = \frac{y^u}{b}$, $\bar{y}^l = \frac{y^l}{b}$ are denoted.

The sum of expressions (4) represents the coefficient of the longitudinal moment generated by the wingtips.

$$M_{zz} = -2\bar{y}^u C^u \cos\varphi \sin\alpha_z + 2\bar{y}^l C^l \cos\varphi \sin\beta , \qquad (5)$$

where m_{zz} is the coefficient of the longitudinal moment generated by the wingtips. The first letter of the index denotes the coordinate axis along which the moment is calculated, while the second letter indicates the aerodynamic wingtips to which the moment magnitude belongs. Let us consider some properties of the coefficients of the aerodynamic forces of the tips, taking into account that the wing tips have the main purpose - to reduce the inductive drag of the wing. Calculations show that the coefficient of inductive resistance is proportional to the square of the lift coefficient.

The coefficient of the longitudinal moment of the aircraft consists of the coefficients of the longitudinal moments of the wing, horizontal tail, fuselage, nacelle, power plants, etc:

$$m_z = m_{z,w} + m_{z,ht} + m_{z,f} + m_{z,n} + m_{z,pp} + \ldots$$

When the coefficients for each component of the aircraft are summed [4,5]

$$m_z = m_{zo} + (\bar{x}_T - \bar{x}_F)C_y + m_z^\delta \delta_{ed} + \ldots \qquad (6)$$

where δ_{ed} is the angle of the elevator deflection,

$$m_{zo} = m_{zo,w} + m_{zo,ht} + m_{zo,f} + \ldots$$

– the coefficient of the longitudinal moment of the aircraft with zero lifting force and with steady RS,

$$\overline{x}_F = \overline{x}_{F,w} + \overline{x}_{F,ht} + \overline{x}_{F,f} + \ldots$$

– the relative coordinate (related to the chord b) of the aerodynamic focus of the aircraft, taking into account its displacement, due to the influence of other parts of the aircraft,

$\overline{x}_T = \dfrac{x_T}{b}$ – the alignment of the aircraft (the relative coordinate of the center of gravity of the aircraft),

$\left(\overline{x}_T - \overline{x}_F\right)$ – the centering reserve.

Note that the relative location of the center of gravity and focus, i.e.

Note that the relative location of the center of gravity and focus, i.e. The reserve of centering is decisive in the formation of the moment ensuring the stability of the aircraft.

For simplicity, we will consider only the terms included in the longitudinal moment coefficient of the aircraft in formula (6). The coefficient of the longitudinal moment for the aircraft, with the presence of aerodynamic wingtips, is then obtained by summing formulas (5) and (6)

$$\hat{m}_z = m_{zo} + \left(\overline{x}_T - \overline{x}_F\right)C_y - 2\overline{y}^u C^u \cos\varphi \sin\alpha_z + 2\overline{y}^l C^l \cos\varphi \sin\beta + m_z^{\delta}\delta_{ed} \quad (7)$$

where the coefficient of the longitudinal moment for an aircraft with a wing having wingtips is indicated by the same symbol for a wing without an end, but is equipped with a cap from above, $\hat{m}_z$. As can be seen, the coefficient of the longitudinal moment for an aircraft with a wing without a tip depends linearly on the lift coefficient, and, as we shall see below, for a aircraft with a wing with wingtips, this dependence is nonlinear.

Conclusions. The aerodynamic forces created by the "Winglet" wingtips during flight have been determined and an expression of the longitudinal moment coefficient and its coefficient has been obtained.

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