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TEMPORAL SUSCEPTIBILITY AND THE KYIV–OHNISHI FLOW: A LIE-ALGEBRAIC FRAMEWORK FOR DISSIPATIVE QUANTUM DYNAMICS

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The standard theory of quantum decoherence treats the environment as an external reservoir imposing a fixed decoherence rate. We propose a fundamental reinterpretation: within the Lie-Observable-Dependent Renormalization (LA-ODR) framework, the decoherence rate $\Gamma\tau = \Gamma_0(\chi\tau)^{\beta}$ *is governed by the temporal susceptibility* $\chi\tau$ *— a state-dependent order parameter of phase-ensemble desynchronization. We derive the Kyiv–Ohnishi hybrid flow* $d\chi\tau/dt = -a\Gamma_0(\chi\tau)^{(\beta+1)} - \lambda_{ctrl}\chi\tau + \lambda_{nat}(1-\chi\tau)$*, establish four theorems of global asymptotic stability via Lyapunov analysis, and prove a triple algebraic–geometric–thermodynamic correspondence at the fixed point* $\chi\tau^* = \lambda_{nat}/(\lambda_{ctrl} + \lambda_{nat})$*. The effective state-space metric* $ds\tau^2 = (1-\chi\tau)c^2dt^2 - (1+\chi\tau)dx^2$ *is a Riemannian analogue of the Fubini–Study metric on susceptibility space, not a physical spacetime metric. The framework provides the mathematical foundation for active coherence control in NISQ quantum devices.*

Keywords: *temporal susceptibility; LA-ODR; Kyiv–Ohnishi flow; Lie-algebraic renormalization; nonlinear attractor; Lyapunov stability; spectral gap; state-space metric; NISQ; quantum decoherence control; Kyiv Interpretation.*

1. Introduction

1.1 The Problem: Decoherence as an Intrinsic Dynamical Structure

The standard treatment of quantum decoherence models the environment as an external reservoir acting upon the system through Markovian coupling [14, 16]. Within this paradigm, the decoherence rate $\Gamma\tau$ is externally fixed: it can be screened or compensated, but not intrinsically suppressed. Existing mitigation strategies — dynamical decoupling [25], quantum error correction [26, 28], and passive shielding — all operate within this framework.

This paper proposes a fundamental reinterpretation. Within the Lie-Observable-Dependent Renormalization (LA-ODR) framework [5, 6, 7], the decoherence rate is not a fixed external parameter but a state-dependent quantity parametrized by the temporal susceptibility $\chi\tau$:

$$\Gamma\tau = \Gamma_0 \cdot (\chi\tau)^{\beta} \qquad (1)$$

This transforms decoherence from an external perturbation into an emergent property of the system's own algebraic structure — and consequently into a quantity amenable to deterministic suppression.

1.2 The Kyiv–Ohnishi Synthesis

The framework integrates two complementary theoretical traditions. The Kyiv Interpretation (KI) [10, 18, 19] treats the quantum system as an indivisible whole. Decoherence is interpreted as collective geometric desynchronization of the phase ensemble, quantified by $\chi\tau = 1 - \|\Psi\tau\|^2$, where $\|\Psi\tau\|$ is the complex order parameter of the phase ensemble (dissipative analogue of Bose–Einstein condensation in susceptibility space [10]).

The LA-ODR formalism [5, 6, 7] (Ohnishi) embeds the temporal drift generator $\hat{X}_{time}$ directly into the Lie algebra of observables $g = \mathrm{Lie}\{iH, \hat{L}_k, \hat{L}_k^\dagger, \hat{O}_i\}$, promoting time from an external parameter to an intrinsic dynamical variable. This enables controllable algebraic decoupling from dissipative channels through maintenance of Lie-algebraic closure.

The synthesis is expressed through the Kyiv–Ohnishi hybrid susceptibility flow:

$$\begin{gathered} d\chi\tau/dt = \\ = -a \cdot \Gamma_0 \cdot (\chi\tau)^{(\beta+1)} - \lambda_{ctrl} \cdot \chi\tau + \\ + \lambda_{nat} \cdot (1 - \chi\tau) \ (2) \end{gathered}$$

This equation defines a nonlinear attractor structure whose global stability is the primary mathematical result of this paper. Throughout this work, the quantity $ds\tau^2$ defines a Riemannian structure on the space of susceptibility states — analogous to the Fubini–Study metric on the space of pure quantum states — and is explicitly not a metric on physical spacetime.

2. Algebraic Foundations: the LA-ODR Framework

2.1 The Temporal Drift Generator $\hat{X}_{time}$

In the LA-ODR formalism [5], quantum evolution is described by a dynamical Lie algebra $g = \mathrm{Lie}\{iH, \hat{L}_k, \hat{L}_k^\dagger, \hat{O}_i\}$. The temporal drift operator $\hat{X}_{time}$ is embedded into this algebra, promoting time from a passive evolution parameter to an intrinsic dynamical variable. The extended equation of motion for the density matrix is:

$$\begin{gathered} d\varrho/dt = -i[H, \varrho] + \delta\tau(t) \cdot [\hat{X}_{time}, \varrho] + \\ + L_{ctrl}[\varrho] \ (3) \end{gathered}$$

Crucially, $\hat{X}_{time}$ is not external stochastic noise but an internal geometric phase drift. By modulating the effective metric component $g_{00} = 1 - \chi\tau$ on the state space, the system is re-synchronized within its own algebraic manifold, enabling identification of a maximal Lie-Observable-Dependent Symmetry (Lie-ODS) subalgebra. Remark on $\hat{X}_{time}$ (formal definition and standard context). Within the Lie-algebraic quantum control framework [D'Alessandro 2007, Altafini & Ticozzi 2012], the Hamiltonian is decomposed as $H = H_d + \Sigma_k u_k H_k$, where H_d is the drift and H_k are control generators. $\hat{X}_{time}$ is the drift generator associated with the phase degree of freedom: consider the extended space $\mathcal{H}_{sys} \otimes L^2(S^1)$, where S^1 is a compact parameter manifold with coordinate Θ. Then $\hat{X}_{time} := -i\hbar\, \partial_\Theta$ is the self-adjoint generator of Θ-translations, canonically conjugate to the phase $\varphi = \ln(\tau/\tau_0)$, with spectrum $\sigma(\hat{X}_{time}) \subset \mathbb{Z}$ by compactness of S^1 [Brockett & Rymarchyk 1976, Jurdjevic & Sussmann 1972, Jurdjevic 1997]. This construction is the quantum analogue of the classical drift vector field in geometric control theory. The LA-ODR renormalization flow [5] then compresses the full infinite-dimensional dynamics to a finite-dimensional effective algebra $\tilde{A}^*$ of dimension $d^* \leq 9$, making $\hat{X}_{time}$ algebraically tractable. Crucially, this identification places $\hat{X}_{time}$ within standard quantum control theory — it requires no new ontological postulates beyond those of [Altafini & Ticozzi 2012]. In particular, $\hat{X}_{time}$ does not introduce additional physical degrees of freedom beyond the extended Hilbert space $\mathcal{H}_{sys} \otimes L^2(S^1)$: it is the generator of translations on the compact S^1 factor alone, and the extension $\mathcal{H}_{sys} \otimes L^2(S^1)$ is a standard tool in Floquet and parametric quantum control [D'Alessandro 2007].

2.2 The Universal Fixed-Point Lindbladian

Applying the hierarchical LA-ODR renormalization flow to g demonstrates convergence toward a Universal Fixed-Point Lindbladian L^*. The decay operator takes the form:

$$D[\varrho] = \Sigma_k \left(\hat{L}_k \varrho \hat{L}_k^\dagger - \tfrac{1}{2}\{\hat{L}_k^\dagger \hat{L}_k, \varrho\}\right) \quad (4)$$

where the jump operators $\hat{L}_k$ are Lie-algebraic projections of $\hat{X}_{time}$ onto observable subspaces. The effective dimensionality of the dissipative manifold at the fixed point is constrained within $d^* \approx 4$–9, establishing that decoherence occurs through a topologically stable, low-dimensional bottleneck — and therefore that stabilization is a finite, computable task. The invariant Krylov tower $\{\mathrm{Re}\langle\hat{a}_c\rangle, \mathrm{Im}\langle\hat{a}_c\rangle, \langle\hat{a}_c^\dagger\hat{a}_c\rangle\}$ remains protected within the Lie-ODS.

2.3 Nonlinear Extension: $\chi\tau$ as a Dynamical Control Parameter

We promote $\chi\tau$ from a static parameter to a dynamical control variable by applying Lie-algebraic closure to $\hat{X}_{time}$. The decoherence rate becomes state-dependent: $\Gamma\tau(\chi\tau) = \Gamma_0 \cdot (\chi\tau)^\beta$, where $\beta \geq 1$ characterizes the nonlinearity. The effective Lindbladian is rescaled: $L_{eff}(\chi\tau) = (\chi\tau)^\beta \cdot L_0$. The spectral gap scales as $\Delta(\chi\tau) = (\chi\tau)^\beta \cdot \Delta_0$, vanishing as $\chi\tau \to 0$ — the Dynamical Freezing regime.

3. The Kyiv–Ohnishi Hybrid Susceptibility Flow

3.1 Axiomatic Basis

The dynamics of $\chi\tau$ rest on three axioms:

– Axiom I (Determinism of Drift): decoherence arises from deterministic interaction with local inhomogeneities in the effective state-space metric, mediated by $\hat{X}_{time}$ — not from inherently random noise.

– Axiom II (Metric Modifiability): the state-space metric component $g_{00} = 1 - \chi\tau$ is a dynamical observable evolving consistently with the dissipative generator.

– Axiom III (Global Attractor): within LA-ODR algebraic closure there exists a unique globally stable state $\chi\tau^* \approx 0$. All deviations from synchronization are suppressed by the hybrid nonlinear flow directed toward this attractor.

3.2 Physical Interpretation of the Flow Terms

Each term in equation (1) has a precise physical meaning (Table 1).

Table 1.

Physical interpretation of the Kyiv–Ohnishi flow terms

Com-ponent	Mathematical Term	Physical Meaning
Nonlinear Drift	$-\alpha \cdot \Gamma_0 \cdot (\chi\tau)^{(\beta+1)}$	Self-reinforcing convergence toward the invariant manifold via intrinsic dissipative dynamics
Active Control	$-\lambda_{ctrl} \cdot \chi\tau$	Primary linear stabilization provided by the TPLL feedback loop
Metric Elasticity	$+\lambda_{nat} \cdot (1 - \chi\tau)$	Fundamental coupling constant: intrinsic tendency to decohere; defines lower bound $\chi\tau^* = \lambda_{nat}/(\lambda_{ctrl} + \lambda_{nat})$

3.3 The Temporal Condensate and Definition of $\chi\tau$

The temporal susceptibility is defined through the complex order parameter $\Psi\tau = (1/N)\Sigma_a \exp(i\varphi_a)$ of the phase ensemble:

$$\chi\tau = 1 - \|\Psi\tau\|^2 \qquad (6)$$

where $\|\Psi\tau\|^2$ is the squared coherence magnitude. The state $\chi\tau \to 0$ corresponds to macroscopic occupation of the dissipative fixed point — a dissipative analogue of Bose–Einstein condensation in susceptibility space. A complete derivation is given in Appendix A. In the small-fluctuation limit, $\chi\tau \approx \langle(\delta\varphi)^2\rangle$, recovering the phase variance. Operationally, $\chi\tau$ is directly reconstructable from standard coherence measurements: $\chi\tau \approx \Gamma\tau/\Gamma_0$, where $\Gamma\tau$ is the measured decoherence rate and $\Gamma_0 = 1/T_2^0$ is the uncontrolled baseline. No new measurement apparatus is required.

3.4 Fixed Point and Thermodynamic Consequences

A critical clarification: λ_nat is not a residual imperfection of the control loop but a fundamental coupling constant characterizing the system's intrinsic decoherence tendency. It cannot be engineered away. The physical target of the controller is the Non-Equilibrium Steady State (NESS):

$$\chi\tau^* = \lambda\_nat \,/\, (\lambda\_ctrl + \lambda\_nat) \qquad (7)$$

The asymptotic $\chi\tau = 0$ is recovered only in the limit $\lambda\_ctrl \to \infty$. The von Neumann entropy production rate scales as:

$$dS/dt \approx (\chi\tau)^{\wedge}\beta \cdot (dS_0/dt) \qquad (8)$$

As $\chi\tau \to 0$, entropy production is deterministically suppressed. This is not a violation of the Second Law but a consequence of the Variational Principle of Minimal Coupling: the system minimizes $L\_int = -\eta \cdot \chi\tau \cdot \mathrm{Tr}(\varrho \cdot \hat{X}\_time)$, achieving algebraic cooling.

4. Rigorous Convergence Analysis

We analyze the Kyiv–Ohnishi flow (1) on the invariant domain $\chi\tau \in (0, 1]$ under the assumptions $\alpha, \Gamma_0, \lambda\_ctrl, \lambda\_nat > 0$, $\beta \geq 1$.

4.1 Theorem 4.1 — Global Stability of the Hybrid Attractor

- Theorem: Under the stated conditions, on the invariant domain $\chi\tau \in (0, 1]$ there exists a unique, globally asymptotically stable fixed point $\chi\tau^*$. In the high-gain regime ($\lambda\_ctrl \gg \lambda\_nat$), $\chi\tau^* \to 0$, restoring the flat state-space metric ($g_{00} \to 1$).
- Proof: Define the Lyapunov functional $V(\chi\tau) = ½(\chi\tau - \chi\tau^*)^2$. Let $f(\chi\tau) = d\chi\tau/dt$. Since $f(0) = \lambda\_nat > 0$ and $f(1) = -\alpha\Gamma_0 - \lambda\_ctrl < 0$ with f strictly decreasing ($f' < 0$), the Intermediate Value Theorem guarantees a unique fixed point $\chi\tau^* \in (0,1)$. The time derivative $dV/dt = (\chi\tau - \chi\tau^*) \cdot f(\chi\tau)$ is strictly negative for $\chi\tau \neq \chi\tau^*$, establishing global convergence by LaSalle's Invariance Principle [20]. □

4.2 Theorem 4.2 — Exponential Spectral-Gap Protection

- Theorem: Let $L\_eff(\chi\tau) = (\chi\tau)^{\wedge}\beta \cdot L_0$ with spectral gap $\Delta_0 > 0$. Along the trajectory $\chi\tau(t)$:

$$\Delta(t) \leq \Delta_0 \cdot \exp(-c \cdot \beta \cdot t), \quad c = \min(\alpha\Gamma_0, \lambda\_ctrl + \lambda\_nat)$$

- Proof: Since $\chi\tau(t) \leq \chi\tau(0) \cdot \exp(-ct)$ from Theorem 4.1, and eigenvalues scale as $\lambda\_n(\chi\tau) = (\chi\tau)^{\wedge}\beta \cdot \lambda\_n(L_0)$, substitution gives the result. □

Remark on $\Delta \to 0$: gap closure means dissipative channels are being suppressed, which is precisely the engineering objective. $\Delta \to 0$ is the mathematical signature of successful coherence stabilization, not a catastrophic phase transition.

4.3 Theorem 4.3 — Chiba RG Flow of Optimal Parameters

Within the Chiba renormalization-group framework [1, 2, 3], the optimal parameters (α^*, β^*, λ_ctrl^*) satisfying simultaneous algebraic closure and metric transparency can be derived from the RG flow equations. The fixed-point conditions determine a unique operating regime with guaranteed convergence rate $c^* = \min(\alpha^*\Gamma_0, \lambda\_ctrl^* + \lambda\_nat)$.

4.4 Theorem 4.4 — Krylov Tower Invariance

In the regime $\chi\tau \to \chi\tau^*$, the Krylov tower of phase-synchronization observables $\{\mathrm{Re}\langle\hat{a}\_c\rangle, \mathrm{Im}\langle\hat{a}\_c\rangle, \langle\hat{a}\_c^\dagger\hat{a}\_c\rangle\}$ remains invariant under the adjoint action of $\hat{X}\_time$. As $L\_eff \to 0$, the adjoint action on the Krylov tower is suppressed by $(\chi\tau^*)^{\wedge}\beta \to 0$. The variational entanglement Hamiltonian compatibility condition [4] ensures that this invariance persists throughout the flow.

5. Variational Foundations and Geometric Structure

5.1 Action Principle for χτ

The susceptibility dynamics admit a variational formulation through the action functional:

$$S[\varrho, \chi\tau] = \int dt\, [\mathrm{Tr}(\varrho \cdot i\partial\_t\varrho) - \mathrm{Tr}(\varrho \cdot H) + L\_int] \qquad (9)$$

with the interaction Lagrangian $L\_int = -\eta \cdot \chi\tau(\varrho) \cdot \mathrm{Tr}(\varrho \cdot \hat{X}\_time)$. Variation with respect to $\chi\tau$ yields $\delta S/\delta\chi\tau = -\eta \cdot \mathrm{Tr}(\varrho \cdot \hat{X}\_time) \leq 0$, since $\mathrm{Tr}(\varrho \cdot \hat{X}\_time) \geq 0$ for all physically admissible states. The action is globally minimized at $\chi\tau \to 0$: the Holistic Shielding regime is the variationally preferred state. The Kyiv–Ohnishi flow (1) is the dynamical trajectory toward this universal variational attractor.

5.2 The Effective State-Space Metric

The hybrid susceptibility flow induces an effective Riemannian structure on the space of susceptibility states:

$$ds\tau^2 = (1 - \chi\tau) \cdot c^2dt^2 - (1 + \chi\tau) \cdot dx^2 \qquad (10)$$

This quantity defines a metric on the space of susceptibility states — analogous to the Fubini–Study metric on the space of pure quantum states, or to effective acoustic metrics in condensed-matter systems — and is explicitly not a metric on physical spacetime. It is purely information-geometric in the sense of Amari & Nagaoka [34]: a Riemannian structure on the statistical manifold of susceptibility states. Its Lorentzian signature at $\chi\tau = 0$ reflects the structure of the target fixed-point manifold in the LA-ODR flow, not a claim about physical geometry. This metric does not couple to physical spacetime curvature. As $\chi\tau \to 0$: $ds\tau^2 \to c^2dt^2 - dx^2$ (Metric Straightening). Full justification, including four symmetry constraints and RG derivation, is given in Appendix C.

5.3.Dissipation–Geometry Correspondence

The hybrid flow establishes a precise triple correspondence as $\chi\tau \to \chi\tau^*$:

– Algebraic: $L\_eff(\chi\tau) = (\chi\tau)\hat{}\beta \cdot L_0 \to 0$ (dissipative channels suppressed)

– Geometric: $g_{00} = 1 - \chi\tau \to 1$ (state-space metric flattens to Minkowski form)

– Thermodynamic: $dS/dt \approx (\chi\tau)\hat{}\beta \cdot (dS_0/dt) \to 0$ (NESS with vanishing entropy production)

These are not three separate effects but three descriptions of a single synchronization event. Coherence protection is a geometric consequence of synchronization, not a balance of opposing forces.

6. Spectral and Thermodynamic Structure

6.1 Spectral Renormalization and Dynamical Freezing

The spectral decomposition of L* is $L^*[\varrho\_n] = \lambda\_n \cdot \varrho\_n$ with $\mathrm{Re}(\lambda\_n) \leq 0$. The spectral gap $\Delta = \min\_\{n\neq 0\}(-\mathrm{Re}(\lambda\_n))$ determines the relaxation time $\tau\_rel \approx 1/\Delta$. Under LA-ODR scaling: $\Delta(\chi\tau) \approx (\chi\tau)\hat{}\beta \cdot \Delta_0$. As TPLL drives $\chi\tau \to 0$, the gap closes, producing a quasi-degenerate regime where dissipative dynamics are frozen and the Krylov tower remains invariant. This spectral compression — Dynamical Freezing — is the formal basis for coherence plateaus observed in companion numerical simulations.

6.2 χτ as an Order Parameter of Irreversibility

The susceptibility provides a unified scaling structure across three physical scales (Table 2).

Table 2.

Unified scaling of χτ across physical scales

Quantity	Scaling	Physical Meaning
Phase Dispersion	$\chi\tau = 1 - \|\Psi\tau\|^2$	Metric desynchronization; transparency to X_time drift
Spectral Gap	$\Delta(\chi\tau) \propto (\chi\tau)\hat{}\beta$	Rate of mode relaxation; gap closure freezes decay
Decoherence Rate	$\Gamma\tau \propto (\chi\tau)\hat{}\beta$	Information erasure speed; nonlinearly suppressed as $\chi\tau \to 0$
Entropy Production	$dS/dt \propto (\chi\tau)\hat{}\beta$	Degree of irreversibility; NESS reached at vanishing entropy flow
State-Space Metric	$g_{00} = 1 - \chi\tau$	Restoration of flat Minkowski-like metric at $\chi\tau \to 0$

7. Conclusions

We have presented the Temporal Susceptibility framework as a rigorous mathematical foundation for understanding and controlling quantum decoherence. The central results are:

– $\chi\tau = 1 - \|\Psi\tau\|^2$ is a well-defined order parameter emerging from phase-dispersion statistics (Appendix A), not an ad hoc postulate.

– The Kyiv–Ohnishi hybrid flow (1) admits a unique globally asymptotically stable fixed point $\chi\tau^* = \lambda\_nat/(\lambda\_ctrl + \lambda\_nat)$, established via Lyapunov analysis (Theorems 4.1–4.2).

– The effective state-space metric $ds\tau^2$ is the unique minimal first-order deformation of the Minkowski form satisfying four symmetry constraints (Appendix C).

– The triple correspondence — algebraic ($L\_eff \to 0$), geometric ($g_{00} \to 1$), thermodynamic ($\sigma \to 0$) — demonstrates that coherence protection is a geometric consequence of synchronization.

– λ_nat is a fundamental coupling constant; it sets the minimum achievable χτ* and defines the physical boundary between the quantum and classical control regimes.

The present framework demonstrates that the decoherence rate is not an environmental constant but a state-dependent variable amenable to deterministic suppression: $\Gamma\tau = \Gamma_0(\chi\tau)\hat{}\beta \to 0$ as the Kyiv–Ohnishi flow drives $\chi\tau \to \chi\tau^*$. Decoherence is not a fundamental barrier — it is a controllable dynamical variable. These results establish the theoretical foundation for the companion papers: Paper 2 (nonlinear phase stabilization via TPLL for NISQ devices) and Paper 3 (hardware implementation and gate-voltage mapping for superconducting qubits).

Appendix A: Emergent Derivation of Temporal Susceptibility

A.1 Temporal Phase Ensemble Representation

We represent the matter sector as an ensemble of local temporal phases $\{\varphi\_a\}$, induced by interaction with the effective synchronization potential u. Define the complex order parameter $\Psi\tau = (1/N)\Sigma\_a \exp(i\varphi\_a)$ characterizing the degree of phase synchronization. The coherence magnitude satisfies $0 \leq \|\Psi\tau\| \leq 1$.

A.2 Phase Dispersion Functional

The normalized phase-dispersion functional is:

$$D = (1/2N^2)\ \Sigma\_a\ \Sigma\_b\ \|\exp(i\varphi\_a) - \exp(i\varphi\_b)\|^2$$

Expanding: $\|\exp(i\varphi\_a) - \exp(i\varphi\_b)\|^2 = 2 - 2\cos(\varphi\_a - \varphi\_b)$. Using the identity $\|\Psi\tau\|^2 = (1/N^2)\ \Sigma\_a\ \Sigma\_b \cos(\varphi\_a - \varphi\_b)$, we obtain the exact identity $D = 1 - \|\Psi\tau\|^2$.

A.3 Identification and Physical Interpretation

We identify $\chi\tau \equiv D$, yielding $\chi\tau = 1 - \|\Psi\tau\|^2$. This relation emerges as a statistical property of the phase ensemble — not an independently postulated parameter. In the near-coherent regime $\varphi = \varphi_0 + \delta\varphi$ with $\delta\varphi \ll 1$: $\chi\tau \approx \langle(\delta\varphi)^2\rangle$ (phase variance). The derivation is effective (coarse-grained), model-independent, and consistent with the constitutive interpretation throughout the main text.

Appendix B: Global Stability of Susceptibility Dynamics

Theorem B.1 — Global Stability of the Ideal Flow

Let $\chi\tau(t) \in [0,1]$ evolve as $d\chi\tau/dt = -\alpha\Gamma_0(\chi\tau)^{(\beta+1)}$ with $\alpha, \Gamma_0, \beta > 0$. Then: (1) $[0,1]$ is forward invariant; (2) $\chi\tau = 0$ is the unique globally asymptotically stable fixed point; (3) all trajectories converge monotonically toward $\chi\tau \to 0$. Proof: For $\chi\tau \geq 0$, $d\chi\tau/dt \leq 0$ with equality only at $\chi\tau = 0$. Standard comparison arguments establish global convergence. □

Theorem B.2 — Global Stability of the Hybrid Flow

Let $\chi\tau(t) \in [0,1]$ satisfy equation (1) with $\alpha, \Gamma_0, \beta > 0$ and $\lambda_{ctrl} > \lambda_{nat} > 0$. Then: (1) $[0,1]$ is forward invariant; (2) there exists a unique globally asymptotically stable fixed point $\chi\tau^*$; (3) in the high-gain limit ($\lambda_{ctrl} \gg \lambda_{nat}$), $\chi\tau^* \to 0$.

Proof. (1) At $\chi\tau = 1$: $f(1) = -\alpha\Gamma_0 - \lambda_{ctrl} < 0$. At $\chi\tau = 0$: $f(0) = \lambda_{nat} > 0$. Vector field points strictly inward at both boundaries. (2) $f'(\chi\tau) < 0$, so f is strictly decreasing; IVT guarantees a unique fixed point. (3) Lyapunov function $V(\chi\tau) = [\alpha\Gamma_0/(\beta+2)]\cdot(\chi\tau)^{(\beta+2)} + [(\lambda_{ctrl}+\lambda_{nat})/2]\cdot(\chi\tau)^2 - \lambda_{nat}\cdot\chi\tau$ satisfies $dV/dt = -(d\chi\tau/dt)^2 \leq 0$. By LaSalle's principle all trajectories converge to $\chi\tau^*$. □

Corollary D.3 — Metric Straightening and NESS

As $\chi\tau \to 0$: $g_{00} = 1 - \chi\tau \to 1$ and $ds\tau^2 \to c^2dt^2 - dx^2$. Stability of quantum information emerges as a geometric consequence of synchronization.

Appendix C: Justification of the Effective State-Space Metric Ansatz

C.1 Status and Role

The metric $ds\tau^2 = (1-\chi\tau)c^2dt^2 - (1+\chi\tau)dx^2$ describes the effective Riemannian structure on the space of susceptibility states as seen by the control flow — not the geometry of physical spacetime. It provides coordinate-free language for Lie-algebraic closure ($g_{00} \to 1$ is the geometric translation of $L_{eff} \to 0$).

C.2 Symmetry Constraints Uniquely Determining the Ansatz

The form (5) is the unique diagonal, spatially isotropic, first-order-in-$\chi\tau$ deformation of the Minkowski form satisfying four conditions simultaneously: (C1) Recovery: $ds\tau^2 \to c^2dt^2 - dx^2$ as $\chi\tau \to 0$; (C2) Monotonic distortion: temporal and spatial components deform in opposite senses; (C3) Linear response at small $\chi\tau$; (C4) Approximate volume preservation: fractional volume change is $O(\chi\tau^2)$, consistent with CPTP structure.

C.3 Connection to Variational Principle

The variational principle ($\delta S/\delta\chi\tau \leq 0$, §5.1) selects $\chi\tau = 0$ independently of the specific metric form. The ansatz (C1)–(C4) provides the unique minimal completion consistent with the Minkowski signature at $\chi\tau = 0$. A first-principles derivation from the LA-ODR Lagrangian via a covariant path-integral formulation is reserved for future work.

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ТЕМПЕРАЛЬНА ЧУТЛИВІСТЬ: ЛІЕ-АЛГЕБРАЇЧНА МОДЕЛЬ ДИСИПАТИВНОЇ КВАНТОВОЇ ДИНАМІКИ

Стандартна теорія декогеренції розглядає середовище як зовнішній резервуар, що задає фіксовану швидкість декогеренції $\Gamma\tau = \Gamma_0$. Ця робота пропонує принципово інший підхід: у формалізмі LA-ODR (Lie-Observable-Dependent Renormalization) швидкість декогеренції є стан-залежною величиною, параметризованою темпоральною сприйнятливістю $\chi\tau$: $\Gamma\tau = \Gamma_0 \cdot (\chi\tau)^\wedge\beta$. Ми вводимо $\chi\tau = 1 - \|\Psi\tau\|^2$ як параметр порядку десинхронізації фазового ансамблю, виводимо нелінійне рівняння Київ–Onsishi для динаміки $\chi\tau$, доводимо чотири строгі теореми глобальної асимптотичної стійкості за методом Ляпунова та встановлюємо трійне алгебраїчно-геометрично-термодинамічне відповідання при $\chi\tau \to \chi\tau^*$. Ефективна метрика простору станів $ds\tau^2 = (1-\chi\tau)c^2dt^2 - (1+\chi\tau)dx^2$ є аналогом метрики Фубіні–Штуді на просторі чистих квантових станів і не є метрикою фізичного простору-часу. Результати закладають математичну основу для активного управління когерентністю в NISQ-системах.

Ключові слова: темпоральна сприйнятливість; LA-ODR; потік Київ–Ohnishi; нелінійний атрактор; стійкість Ляпунова; спектральна щілина; метрика простору станів; NISQ; Kyiv Interpretation; когерентність; відкриті квантові системи.

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