

DOI: 10.18372/2310-5461.70.21196

UDC 621.391.8:519.21

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## THEORETICAL FOUNDATIONS OF EQUIVALENT TRANSFORMATION OF UNEQUAL-ENERGY COMPLEX SIGNAL ENSEMBLES FOR CODE DIVISION MULTIPLE ACCESS SYSTEMS

### Introduction

Ultra-wideband access systems with pulse signals create a technical basis for the development of wireless systems with restrictions on energy consumption, electromagnetic compatibility, and operation in high-frequency bands [1, 2]. A promising direction in the development of ultra-wideband access systems is the transition to code division multiple access based on ensembles of complex pulse signals. In [3, 4], ensembles of unequal-energy complex pulse signals are considered, which are capable of providing a limited level of multiple access interference under arbitrary time shifts. In contrast to equal-energy signal ensembles, for which the relations for efficiency evaluation mostly have a simple form suitable for application, the efficiency evaluation of unequal-energy complex signal ensembles usually requires the use of the corresponding relations in general forms. Therefore, the development of the theoretical foundations of the equivalent transformation of unequal-energy complex signal ensembles is relevant, under which the relations for their efficiency evaluation can be reduced to a form similar to that of the equal-energy case.

### Analysis of recent research and publications

In code division multiple access systems, the relation between the levels of useful signal, multiple access interference, and noise is determined through the signal-to-interference-plus-noise ratio (SINR) [5–8]. In energy form, SINR as the ratio of useful signal energy to the sum of multiple access interference energy  $E_{I,i}$  and noise for the  $i$ -th user  $\text{SINR}_i$  is determined by the relation:

$$\text{SINR}_i = \frac{E_i}{E_{I,i} + E_{N,i}},$$

$$E_{I,i} = \sum_{j=1, j \neq i}^L E_j \left( R_{ij}^{(max)} \right)^2,$$

where  $E_i$  – useful signal energy of the  $i$ -th user;  $E_{I,i}$  – multiple access interference energy for the  $i$ -th user;  $E_{N,i}$  – noise energy for the  $i$ -th user;  $E_j$  – energy of the  $j$ -th interfering signal;  $R_{ij}^{(max)}$  – maximum cross-correlation value for the pair of signals  $i$  and  $j$ ;  $L$  – signal ensemble volume.

In the equal-energy case, after normalization of signal energies to a unit value, the upper estimate of multiple access interference energy  $E_{I,i}^{(max)}$  determined through the maximum ensemble value of the squared cross-correlation  $R_{max}^2$ :

$$E_{I,i}^{(max)} = (L - 1)R_{max}^2.$$

Then we can write:

$$\text{SINR}_j \geq \frac{1}{(L - 1)R_{max}^2 + E_{N,i}},$$

or, using the approximation  $L - 1 \approx L$  if  $L \gg 1$ :

$$\text{SINR}_j \geq \frac{1}{LR_{max}^2 + E_{N,i}}.$$

For ensembles of equal-energy signals, after normalization, the component of the SINR denominator that determines the effect of multiple access interference takes the form of the product  $(L - 1)R_{max}^2$ . The value  $L - 1$  determines the normalized total energy of all interfering signals and therefore is the energy component of the product. The value  $R_{max}^2$  is the correlation component of the product and characterizes the structure of the ensemble signals. For unequal-energy complex signal ensembles, the component of the SINR denominator that determines the effect of multiple access interference

cannot be exactly reduced to the form of the equal-energy case, because the signal energies of the ensemble enter the terms of the sum in the expression for  $E_{j,i}$  as unequal factors. At the same time, a possible way to apply the form of a product of energy and structural components to unequal-energy complex signal ensembles is to use an approximation based on separate accounting of the structural and energy properties of the ensemble. Thus, to preserve the product form characteristic of the equal-energy case, it is necessary to determine a method for approximate reduction of the set of signal energies of the ensemble to a scalar value that can serve as the energy component.

Since the difference between signal energies of the ensemble forms nonuniformity of their distribution, further analysis of the energy component should be performed on the basis of a mathematical apparatus in which the nonuniformity of a finite set of nonnegative values is characterized through the relations between the sum of the values, the sum of their squares, the mean value, and the variance [9, 10], as well as through concentration measures [11–15].

### Problem Statement

The problem consists in the need to obtain an SINR estimate for an unequal-energy complex signal ensemble in the form of a relation obtained for the equal-energy case by approximately representing the component of the SINR denominator that determines the effect of multiple access interference as a product of two numerical values, namely the energy and correlation components, whose value determines the normalized multiple access interference energy in the equal-energy case. For such a representation, it is necessary to determine a method for approximate reduction of the set of signal energies of the ensemble to a scalar value that can serve as the energy component, whereas the correlation component can be the result of approximate reduction of the squared cross-correlations of the ensemble to the maximum ensemble value. In this work, the method for approximate reduction of the set of signal energies of the ensemble to a scalar value that can serve as the energy component determines the equivalent transformation of an unequal-energy complex signal ensemble into an equal-energy signal ensemble.

### The purpose of the article

The purpose of the article is to develop the theoretical foundations of the equivalent transformation of unequal-energy complex signal ensembles for code division multiple access systems, which provides the reduction of the relations for their efficiency evaluation to a form similar to that of the equal-energy case by approximately representing the

component of the SINR denominator that determines the effect of multiple access interference as a product of energy and correlation components.

### Summary of the main material

To pass from the set of signal energies of the ensemble to a scalar energy component, let us consider the form of the Cauchy–Bunyakovsky inequality normalized by the ensemble volume [10]:

$$\left(\frac{1}{L}\sum_{i=1}^L E_i\right)^2 \leq \frac{1}{L}\sum_{i=1}^L E_i^2.$$

The variance of the signal energies of the ensemble  $\sigma_E^2$  is equal to the difference between the right-hand and left-hand sides of the normalized Cauchy–Bunyakovsky inequality:

$$\sigma_E^2 = \frac{1}{L}\sum_{i=1}^L E_i^2 - \left(\frac{1}{L}\sum_{i=1}^L E_i\right)^2.$$

The mean value of the squared signal energies, that is, the right-hand side of the Cauchy–Bunyakovsky inequality, is always greater than or equal to the square of the mean value of the signal energies, that is, the left-hand side of this inequality. Equality is achieved only for equal-energy signal ensembles, for which  $\sigma_E^2 = 0$ .

For further analysis, the total signal energy of the ensemble will be regarded as its energy resource, which does not change under approximate reduction of the set of signal energies of the ensemble to a scalar value. Therefore, only cases of redistribution of energies between the signals of the ensemble will be considered further, when the total signal energy of the ensemble is constant.

Thus, for a given ensemble, the square of the mean signal energy is determined by the total signal energy of the ensemble and its volume, whereas the mean value of the squared signal energies is determined by the signal energy distribution of the ensemble. Therefore, the variance of the signal energies of the ensemble reflects the deviation of the signal energy distribution of the ensemble from the equal-energy case. Any nonuniformity of the signal energy distribution leads to violation of equality between the right-hand and left-hand sides of the Cauchy–Bunyakovsky inequality due to an increase in the mean value of the squared signal energies relative to the square of their mean energy.

Next, let us pass to the non-normalized form of the Cauchy–Bunyakovsky inequality by multiplying the left-hand and right-hand sides of the normalized inequality by the ensemble volume  $L$ :

$$\left(\sum_{i=1}^L E_i\right)^2 \leq L\sum_{i=1}^L E_i^2.$$

It follows from the last relation that, for unchanged total energy of the ensemble and unchanged ensemble volume, the sum of squared signal energies cannot be less than  $(\sum_{i=1}^L E_i)^2/L$ , and equality is achieved only for the equal-energy case. Therefore, the value  $(\sum_{i=1}^L E_i)^2/L$  determines the value of the sum of squared signal energies corresponding to the equal-energy case under unchanged total energy of the ensemble and unchanged ensemble volume.

The relation for the variance of the signal energies of the ensemble can also be written through non-normalized values:

$$L\sigma_E^2 = \sum_{i=1}^L E_i^2 - \frac{(\sum_{i=1}^L E_i)^2}{L}.$$

When  $\sigma_E^2 = 0$ , we have the equal-energy case, for which

$$\sum_{i=1}^L E_i^2 = \frac{(\sum_{i=1}^L E_i)^2}{L}.$$

When  $\sigma_E^2 \neq 0$ , we have the unequal-energy case, for which

$$\sum_{i=1}^L E_i^2 > \frac{(\sum_{i=1}^L E_i)^2}{L}.$$

Thus, in the non-normalized form, the value  $L\sigma_E^2$  determines the excess of the sum of squared signal energies over the value corresponding to the equal-energy case.

For further analysis, let us pass from absolute energies to normalized energy values and, for each signal  $E_i$ , introduce the share of its energy  $p_i$  in the total energy of the ensemble  $E_\Sigma$ :

$$p_i = \frac{E_i}{E_\Sigma}, \quad i = 1, \dots, L, \quad \sum_{i=1}^L p_i = 1,$$

where  $E_\Sigma$  – total signal energy of the ensemble;  $E_\Sigma = \sum_{i=1}^L E_i$ .

Since the values  $p_i = E_i/E_\Sigma$ , are normalized shares of signal energies and satisfy the condition  $\sum_{i=1}^L p_i = 1$ , further analysis should be performed through the sum of squares of these shares  $\kappa_E$ , which, after substitution of the expression for  $p_i$ , takes the form

$$\kappa_E = \sum_{i=1}^L p_i^2 = \sum_{i=1}^L \left( \frac{E_i}{\sum_{j=1}^L E_j} \right)^2 = \frac{\sum_{i=1}^L E_i^2}{(\sum_{i=1}^L E_i)^2}.$$

The dimensionless value  $\kappa_E$  is a quadratic measure of concentration of the signal energies of the ensemble, hereinafter concentration [11–15], which is determined only by the signal energy distribution of the ensemble and does not change when all energies are multiplied by a common coefficient. For

an equal-energy signal ensemble, the concentration  $\kappa_E$  takes its minimum value, and when all energy is concentrated in one signal, it takes its maximum value.

Now let us pass from the concentration  $\kappa_E$  to a value that directly reflects the deviation of the distribution of energy shares from the equal-energy case. For an equal-energy signal ensemble, all energy shares are identical and equal to  $1/L$ , which gives zero deviation for the normalized distribution of shares. Therefore, the variance of the normalized signal energy shares  $\sigma_p^2$  is determined relative to the equal-energy case:

$$\sigma_p^2 = \frac{1}{L} \sum_{i=1}^L \left( p_i - \frac{1}{L} \right)^2.$$

For an equal-energy signal ensemble, all shares coincide with the value  $1/L$ , so  $\sigma_p^2 = 0$ . For an unequal-energy signal ensemble, the signal energy distribution becomes nonuniform, the shares deviate from the value  $1/L$ , and therefore  $\sigma_p^2$  increases. Thus, the variance  $\sigma_p^2$  reflects the measure of deviation of the distribution of energy shares from the equal-energy case.

Let us establish the relation between the variance  $\sigma_p^2$  and the concentration  $\kappa_E$  in order to separate the property of deviation of the distribution of shares from the equal-energy case from the property of energy concentration in individual signals of the ensemble. To do this, expand the square in the relation for  $\sigma_p^2$  and use the normalization condition for the shares  $\sum_{i=1}^L p_i = 1$ :

$$\sigma_p^2 = \frac{1}{L} \sum_{i=1}^L p_i^2 - \frac{1}{L^2},$$

after which we substitute  $\sum_{i=1}^L p_i^2 = \kappa_E$  and obtain the relation

$$\sigma_p^2 = \frac{\kappa_E}{L} - \frac{1}{L^2},$$

which shows that, for a given ensemble volume  $L$ , the variance of the normalized shares  $\sigma_p^2$  and the concentration  $\kappa_E$  are related one-to-one. In this case, the variance  $\sigma_p^2$  characterizes the deviation of the distribution of shares from the equal-energy case, whereas the concentration  $\kappa_E$  characterizes the degree of energy concentration in individual signals of the ensemble. Thus, the same nonuniformity of the energy distribution can be represented either in the form of variance or in the form of concentration. Solving the last relation with respect to  $\kappa_E$ , we can write

$$\kappa_E = L\sigma_p^2 + \frac{1}{L}.$$

Thus, the values  $\kappa_E$  and  $\sigma_p^2$  reflect the same property of nonuniformity of the energy distribution, but in different mathematical forms. The value  $\sigma_p^2$  expresses the nonuniformity of the energy distribution through the squares of deviations of the energy shares from the equal-energy case and is therefore a measure of deviation of the distribution from the uniform one, whereas the value  $\kappa_E$  expresses this nonuniformity through the sum of squares of the energy shares and is therefore a measure of concentration of the signal energy distribution of the ensemble.

For further consideration of the energy component, not only the one-to-one relation between  $\kappa_E$  and  $\sigma_p^2$  is essential, but also the meaning of the increase in  $\kappa_E$  under reduction of the set of signal energies of the ensemble to a scalar value. Since, in an unequal-energy ensemble, signal energies enter the sum of multiple access interference energy as unequal energy factors, the increase in  $\kappa_E$  means that the distribution of the total energy of the ensemble shifts toward signals with higher energies, whereas the contribution of signals with lower energies to the total multiple access interference energy relatively decreases. Due to such redistribution, it is the signals with higher energies that begin to determine the main contribution to the total multiple access interference energy and, accordingly, to the energy component. Therefore, the increase in  $\kappa_E$  corresponds to strengthening of the signal dominance effect in the formation of the energy component, and the value  $\kappa_E$  should be interpreted as a scalar measure of manifestation of this effect.

After this interpretation of the concentration  $\kappa_E$ , let us establish the bounds of its possible values. In the equal-energy case, dominance is absent, so  $\kappa_E$  takes its minimum value  $1/L$ .

As dominance increases, the value  $\kappa_E$  increases, and in the limiting case, when the entire total energy belongs to one signal, it takes the value 1.

Thus, the variance  $\sigma_p^2$  interprets the nonuniformity of the energy distribution as a measure of deviation from the equal-energy case, whereas the concentration  $\kappa_E$  interprets the same nonuniformity as a measure of dominance of signals with higher energies. To establish the bounds of  $\kappa_E$ , apply the Cauchy–Bunyakovsky inequality to the normalized energy shares. Since  $\sum_{i=1}^L p_i = 1$ , the inequality gives the estimate

$$\frac{1}{L} \leq \sum_{i=1}^L p_i^2.$$

After substituting  $\sum_{i=1}^L p_i^2 = \kappa_E$ , we obtain

$$\frac{1}{L} \leq \kappa_E,$$

where equality is achieved only for the equal-energy case, when all signal energies are identical. Thus, the lower bound of concentration corresponds to the minimum possible nonuniformity of the energy distribution in the ensemble.

The upper bound of the concentration  $\kappa_E$  is found from the conditions  $0 \leq p_i \leq 1$ , which hold for each normalized energy share. Taking into account that the square of each share does not exceed the value of the share itself, we have the upper bound of concentration

$$\sum_{i=1}^L p_i^2 \leq \sum_{i=1}^L p_i = 1, \kappa_E \leq 1,$$

which corresponds to full concentration of the entire energy resource in one signal of the ensemble.

Combining the lower and upper bounds, we obtain the full range of variation of the concentration  $\kappa_E$  for an ensemble of volume  $L$ :

$$\frac{1}{L} \leq \kappa_E \leq 1.$$

Thus, the lower bound of concentration corresponds to the equal-energy case, when energy is distributed equally among all signals, whereas the upper bound corresponds to concentration of the entire energy resource in one signal of the ensemble. As the concentration  $\kappa_E$  increases, the nonuniformity of the energy distribution in the ensemble increases.

Now let us pass to the value  $\kappa_E^{-1}$ , inverse to the concentration  $\kappa_E$ :

$$\kappa_E^{-1} = \frac{1}{\kappa_E},$$

for which the bounds of values are obtained from the previously established bounds for the concentration  $\kappa_E$ :

$$\frac{1}{L} \leq \kappa_E \leq 1.$$

Since  $\kappa_E > 0$ , when passing to the inverse value, the inequality is reversed, and therefore we have

$$1 \leq \kappa_E^{-1} \leq L.$$

The lower bound for  $\kappa_E^{-1}$  corresponds to the case of full concentration of the total signal energy of the ensemble in one signal, when  $\kappa_E = 1$  and therefore  $\kappa_E^{-1} = 1$ . The upper bound corresponds to the equal-energy case, when  $\kappa_E = 1/L$  and therefore  $\kappa_E^{-1} = L$ . Thus, the bounds of the value  $\kappa_E^{-1}$  follow from the previously established bounds of the direct value  $\kappa_E$ .

It should also be noted that the product of the values  $\kappa_E$  and  $\kappa_E^{-1}$  is equal to one:

$$\kappa_E \kappa_E^{-1} = 1,$$

which expresses their reciprocal nature.

To represent the nonuniformity of the energy distribution relative to the equal-energy case, let us

normalize  $\kappa_E$  and  $\kappa_E^{-1}$  by the values that they take in the equal-energy case, respectively  $1/L$  and  $L$ :

$$\frac{\kappa_E}{1/L} = L\kappa_E,$$

$$\frac{\kappa_E^{-1}}{L} = \frac{1}{L\kappa_E},$$

as a result, we obtain relations for normalized concentration measures, the values of which are equal to one for the equal-energy case:

$$\frac{\kappa_E}{1/L} = \frac{\sum_{i=1}^L E_i^2}{(\sum_{i=1}^L E_i)^2 / L},$$

$$\frac{\kappa_E^{-1}}{L} = \frac{(\sum_{i=1}^L E_i)^2 / L}{\sum_{i=1}^L E_i^2}.$$

For unchanged total energy of the ensemble and unchanged ensemble volume, an increase in nonuniformity of the signal energies of the ensemble leads to an increase in the value  $\sum_{i=1}^L E_i^2$ . Therefore, the normalized value of the concentration measure increases relative to one, whereas the normalized inverse value decreases relative to one.

To introduce a dimensionless second-order measure of nonuniformity, use the square of the coefficient of variation of the signal energies of the ensemble  $C_v^2$ , which is the variance normalized by the square of the mean energy and therefore does not depend on the absolute energy level and expresses only the nonuniformity of the signal energy distribution:

$$\mu_E = \frac{1}{L} \sum_{i=1}^L E_i,$$

$$\sigma_E^2 = \frac{1}{L} \sum_{i=1}^L (E_i - \mu_E)^2 = \frac{1}{L} \sum_{i=1}^L E_i^2 - \mu_E^2,$$

$$C_v^2 = \frac{\sigma_E^2}{\mu_E^2} = \frac{\frac{1}{L} \sum_{i=1}^L E_i^2 - \mu_E^2}{\mu_E^2} = \frac{\frac{1}{L} \sum_{i=1}^L E_i^2}{\mu_E^2} - 1,$$

where  $\sigma_E^2$  – variance of the signal energies of the ensemble;  $\mu_E$  – mean value of the signal energies of the ensemble;  $C_v^2$  – square of the coefficient of variation of the signal energies of the ensemble.

Let us write the expression for the square of the mean signal energy of the ensemble  $\mu_E^2$  and obtain the value  $C_v^2$  determined directly through the signal energies:

$$\mu_E^2 = \left( \frac{1}{L} \sum_{i=1}^L E_i \right)^2 = \frac{(\sum_{i=1}^L E_i)^2}{L^2},$$

$$C_v^2 = \frac{\frac{1}{L} \sum_{i=1}^L E_i^2}{\left( \frac{1}{L} \sum_{i=1}^L E_i \right)^2} - 1 = L \frac{\sum_{i=1}^L E_i^2}{(\sum_{i=1}^L E_i)^2} - 1,$$

where the factor multiplied by  $L$  contains the expression that coincides with the right-hand side of the relation for  $\kappa_E$ :

$$\frac{\sum_{i=1}^L E_i^2}{(\sum_{i=1}^L E_i)^2} = \kappa_E,$$

therefore  $C_v^2 = L\kappa_E - 1$ .

From the obtained relation  $C_v^2 = L\kappa_E - 1$  the dimensionless forms normalized relative to the equal-energy case directly follow:

$$\frac{\kappa_E}{1/L} = L\kappa_E = 1 + C_v^2,$$

$$\frac{\kappa_E^{-1}}{L} = \frac{1}{L\kappa_E} = \frac{1}{1 + C_v^2},$$

the first of which, for the case of unequal-energy signal ensembles, when  $C_v^2 > 0$ , will be greater than one,  $L\kappa_E = 1 + C_v^2$ , and the second will be less than one,  $1/(1 + C_v^2)$ ; for the case of equal-energy signal ensembles, when  $C_v^2 = 0$ , these values will be equal to one.

Thus, the dimensionless forms obtained above represent the deviation of the energy component from the equal-energy case, which allows the result of reducing the set of signal energies of the ensemble to a scalar value to be considered as an equivalent transformation under which the form of the product of the energy and correlation components, characteristic of the equal-energy case, is preserved.

## Conclusions

The theoretical foundations of the equivalent transformation of unequal-energy complex signal ensembles for code division multiple access systems have been developed, which provide the reduction of the relations for efficiency evaluation to a form similar to that of the equal-energy case. An approximate transformation of an unequal-energy complex signal ensemble into a notional equal-energy complex signal ensemble equivalent in terms of the total signal energy has been determined. This transformation transfers the effect of nonuniform signal energy distribution on multiple access interference into a change in the volume of the equal-energy signal ensemble at a constant value of the maximum ensemble cross-correlation by means of approximate reduction of the set of signal energies to the scalar value of the volume of the equivalent equal-energy complex signal ensemble. To implement the equivalent transformation, two interrelated variants of approximate scalar reduction of the set of signal energies of the ensemble to a scalar value based on mutually reciprocal concentration measures normalized to the equal-energy value have been proposed. These variants have opposite effects on the change in the volume of the equivalent equal-energy

complex signal ensemble and thereby define two different interpretations of the effect of multiple access interference on users with the lowest and highest signal energies, which directly corresponds to different degrees of manifestation of the signal dominance effect.

### REFERENCES

- [1] IEEE Standard for Information technology–Local and metropolitan area networks–Specific requirements–Part 15.4: Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for Low-Rate Wireless Personal Area Networks (WPANs): Amendment 1: Add Alternate PHYs. IEEE 802.15.4a-2007. <https://standards.ieee.org/ieee/802.15.4a/3571/>.
- [2] IEEE Standard for Low-Rate Wireless Networks–Amendment 1: Enhanced Ultra Wideband (UWB) Physical Layers (PHYs) and Associated Ranging Techniques. IEEE 802.15.4z-2020. <https://standards.ieee.org/ieee/802.15.4z/10230/>.
- [3] S. V. Indyk, V. P. Lysechko, O. S. Zhuchenko, and V. S. Kitov, “The Formation Method of Complex Signals Ensembles by Frequency Filtration of Pseudo-Random Sequences with Low Interaction in the Time Domain,” *Radio Electronics, Computer Science, Control*, no. 4, pp. 7–14, 2020, <https://doi.org/10.15588/1607-3274-2020-4-1>.
- [4] V. P. Lysechko, O. S. Zhuchenko, and S. V. Indyk, “Method for Synthesis of Ensembles of Complex Time-Division Signals with a Periodic Structure for Ultra-Wideband Multiple Access Systems,” in *Theory and Practice of Modern Science in Conditions of Transformations, Proceedings of the Scientific and Practical Conference, Poltava, April 24–25, 2026*, Odesa: Molodyi Vchenyi Publishing House, 2026, pp. 142–144.
- [5] B. Sklar, *Digital Communications: Fundamentals and Applications*, 2nd ed., Upper Saddle River: Prentice Hall PTR, 2001, <https://www.informit.com/store/digital-communications-fundamentals-and-applications-9780130847881>.
- [6] V. P. Ipatov, *Spread Spectrum and CDMA: Principles and Applications*, Chichester: John Wiley & Sons, 2005, <https://doi.org/10.1002/0470091800>.
- [7] A. J. Viterbi, *CDMA: Principles of Spread Spectrum Communication*, Reading: Addison-Wesley, 1995, <https://www.ece.ucsd.edu/faculty-research/books-by-faculty/cdma-principles-of-spread-spectrum-communication>.
- [8] S. Verdú, *Multiuser Detection*, Cambridge: Cambridge University Press, 1998, <https://www.cambridge.org/9780521593731>.
- [9] A. Klenke, *Probability Theory: A Comprehensive Course*, 3rd ed., Universitext, Springer, Cham, 2020, <https://doi.org/10.1007/978-3-030-56402-5>.
- [10] J. M. Steele, *The Cauchy-Schwarz Master Class: An Introduction to the Art of Mathematical Inequalities*, Cambridge: Cambridge University Press, 2004, 318 p., <https://doi.org/10.1017/CBO9780511817106>.
- [11] E. H. Simpson, “Measurement of diversity,” *Nature*, vol. 163, p. 688, 1949, <https://doi.org/10.1038/163688a0>.
- [12] M. O. Hill, “Diversity and evenness: A unifying notation and its consequences,” *Ecology*, vol. 54, no. 2, pp. 427–432, 1973, <https://doi.org/10.2307/1934352>.
- [13] L. Jost, “Entropy and diversity,” *Oikos*, vol. 113, no. 2, pp. 363–375, 2006, <https://doi.org/10.1111/j.2006.0030-1299.14714.x>.
- [14] A. Chao, C.-H. Chiu, and L. Jost, “Phylogenetic diversity measures based on Hill numbers,” *Philosophical Transactions of the Royal Society B*, vol. 365, no. 1558, pp. 3599–3609, 2010, <https://doi.org/10.1098/rstb.2010.0272>.
- [15] C.-H. Chiu and A. Chao, “Distance-based functional diversity measures and their decomposition: A framework based on Hill numbers,” *PLoS ONE*, vol. 9, no. 7, Art. no. e100014, 2014, <https://doi.org/10.1371/journal.pone.0100014>.

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### ТЕОРЕТИЧНІ ОСНОВИ ЕКВІВАЛЕНТНОГО ПЕРЕТВОРЕННЯ АНСАМБЛІВ РІЗНОЕНЕРГЕТИЧНИХ СКЛАДНИХ СИГНАЛІВ ДЛЯ СИСТЕМ МНОЖИННОГО ДОСТУПУ З КОДОВИМ РОЗДІЛЕННЯМ

*Перспективним напрямом розвитку надширококузових систем доступу є перехід до застосування множинного доступу з кодовим розділенням на основі ансамблів імпульсних різноенергетичних складних сигналів. На відміну від ансамблів рівноенергетичних сигналів, для яких співвідношення для оцінки ефективності мають переважно просту форму, придатну для застосування, оцінка ефективності різноенергетичних ансамблів складних сигналів, як правило, потребує використання відповідних співвідношень у загальних формах. У статті розроблено теоретичні основи еквівалентного перетворення ансамблів різноенергетичних складних сигналів систем множинного доступу з кодовим розділенням, що забезпечують приведення співвідношень для оцінки їх ефективності до форми, подібної до форми рівноенергетичного випадку. Визначено наближене перетворення ансамблю різноенергетичних складних сигналів в умовний ансамбль рівноенергетичних складних сигналів, еквівалентний за сумарною енергією сигналів, яке переносить вплив нерівномірності розподілу енергій сигналів на завади множинного доступу у зміну об'єму ансамблю рівноенергетичних складних сигналів при сталій величині максимальної взаємної кореляції ансамблю шляхом наближеного зведення сукупності енергій сигналів до скалярної величини об'єму еквівалентного ансамблю рівноенергетичних складних сигналів, наслідком*

якого є приведення співвідношень для оцінки ефективності ансамблів різноенергетичних складних сигналів до форми, подібної до форми рівноенергетичних складних сигналів. Для реалізації еквівалентного перетворення запропоновано два взаємопов'язані варіанти наближеного скалярного зведення сукупності енергій сигналів ансамблю до скалярної величини на основі нормованих до рівноенергетичного значення взаємообернених мір концентрації, які мають протилежний вплив на зміну об'єму еквівалентного ансамблю рівноенергетичних складних сигналів і тим задають дві різні інтерпретації впливу завад множинного доступу на користувачів з найменшими і найбільшими енергіями сигналів, що безпосередньо відповідає різному ступеню прояву ефекту домінування сигналів.

**Ключові слова:** завади множинного доступу, енергія сигналу, еквівалентне перетворення, концентрація, система множинного доступу з кодовим розділенням, різноенергетичний ансамбль складних сигналів, рівноенергетичний ансамбль складних сигналів.

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## **THEORETICAL FOUNDATIONS OF EQUIVALENT TRANSFORMATION OF UNEQUAL-ENERGY COMPLEX SIGNAL ENSEMBLES FOR CODE DIVISION MULTIPLE ACCESS SYSTEMS**

*A promising direction in the development of ultra-wideband access systems is the transition to code division multiple access based on ensembles of unequal-energy complex pulse signals. In contrast to equal-energy signal ensembles, for which the relations for efficiency evaluation mostly have a simple form suitable for application, the efficiency evaluation of unequal-energy complex signal ensembles usually requires the use of the corresponding relations in general forms. The article develops the theoretical foundations of the equivalent transformation of unequal-energy complex signal ensembles for code division multiple access systems, which provide the reduction of the relations for efficiency evaluation to a form similar to that of the equal-energy case. An approximate transformation of an unequal-energy complex signal ensemble into an equivalent equal-energy complex signal ensemble, equivalent in terms of the total signal energy, is determined. This transformation represents the effect of nonuniform signal energy distribution on multiple access interference through a change in the volume of the equivalent equal-energy complex signal ensemble at a constant value of the maximum ensemble cross-correlation. This is achieved by the approximate reduction of the set of signal energies to a scalar value of the volume of the equivalent equal-energy complex signal ensemble, which results in reducing the relations for efficiency evaluation of unequal-energy complex signal ensembles to a form similar to that of equal-energy complex signal ensembles. To implement the equivalent transformation, two interrelated variants of approximate scalar reduction of the set of signal energies of the ensemble to a scalar value are proposed. These variants are based on a concentration measure and its inverse concentration normalized to the equal-energy value. They have opposite effects on the change in the volume of the equivalent equal-energy complex signal ensemble and thereby define two different interpretations of the effect of multiple access interference on users with the lowest and highest signal energies, which directly corresponds to different degrees of manifestation of the signal dominance effect.*

**Keywords:** multiple access interference; signal energy; equivalent transformation; concentration; code division multiple access system; unequal-energy complex signal ensemble; equal-energy complex signal ensemble.

Received: 10.05.2026 p.

Accepted: 15.05.2026 p.

Published: 28.05.2026 p.