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ENERGY LOAD CONTROL OF BRAKES' FRICTION PAIRS

Theoretical research and a computational experiment dedicated to optimal control of thermal processes in friction pairs during frictional interaction in braking devices have allowed us to establish and propose the following. Thermal process control is accomplished mathematically using the heat equation with initial and boundary conditions, employing a control function, as well as the method of thermoelastic potentials and moving local heat sources in brake friction pairs. Material selection based on the mechanical properties of drill drawworks brake pulleys was conducted using a computational experiment. Based on the research conducted, a discovery formula was formulated: "Unknown patterns of microcrack occurrence and development on the working surfaces of metal friction brake elements have been established."

Key words: control, energy loading, braking devices, friction pair, equivalent stresses.

Introduction. Wear and failure of materials during friction are caused by the combined action of surface temperature and a temperature gradient, which induces equivalent stresses in the material. When a temperature gradient occurs in the material of a metal element, or when a material consisting of two or more components with different expansion coefficients is heated, its individual components expand differently in accordance with their surface-volume temperature and failure coefficient. A material's resistance to thermal impulse is understood as its resistance under conditions of instantaneous heat generation. Thus, the problem of resistance to thermal impulse and thermal stresses is reduced to determining the thermal stresses caused by the temperature field of the material of a metal brake friction element.

Analysis of literary sources and the state of the problem. Significant progress in ensuring the desired limit behavior of adaptive control systems with input constraints of the saturation type was achieved in [1]. The authors established a condition under which, in the absence of disturbances, the output error converges to zero, and the control input ceases to saturate after a finite time. New results were obtained by the same authors for the case of bounded disturbances of a certain class in [2]. Thus, the question of under what conditions the desired asymptotic properties of an adaptive control system with bounded input, in particular suboptimality, could be achieved in the case of bounded disturbances has not yet been resolved.

In [3], an adaptive control system containing a discrete linear stationary plant with arbitrary disturbance constraints, the control input of which is bounded within certain limits, is considered. Sufficient conditions are established that guarantee global asymptotic stability and simultaneously suboptimality of operating parameters. In [4-5], it was not proven that the output errors and control sequences converge. It turns out

that although these signals remain bounded, such control systems may not be asymptotically stable, even when the plant, whose parameters are known, is strictly stable and minimum-phase.

To control the heat generation process during friction in drum-shoe brakes [6], it is necessary to consider the influence of various factors on the temperature field of the friction pair. Cooling has a significant effect on the heating depth of the surface layers. With identical heat source parameters, the maximum heating depth of the surface layer to a given temperature with cooling is always less than without cooling.

The aim of this research is to control the energy loading of friction pairs of braking devices through mathematical operations and computational experiments.

Optimal control of thermal processes of friction pairs during frictional interaction. To control the heat generation process during braking, it is necessary to take into account cooling and temperature gradient in the calculation model as possible causes of cracking of the pulley material.

Let us compose a differential equation for the thermal Q balance of heat during braking of friction pairs

$$Qd\tau = cmd\Delta\vartheta + A_2\sigma'\Delta\vartheta d\tau, \quad (1)$$

hence the increment in temperature difference has the form

$$\frac{d\Delta\vartheta}{d\tau} = \frac{1}{cm}(Q - A_2\sigma'\Delta\vartheta). \quad (2)$$

where: $d\tau$ is the change in braking time; c is the specific heat capacity of the metal; m is the mass of the pulley; $\Delta\vartheta$ is the change in surface temperature; A_2 is the matte surface area of the pulley; σ is the heat transfer coefficient.

On the other hand, the heat during braking goes to heating the rim of the brake pulley and is removed in the form of heat exchange, the balance of which can be written as

$$Q - A_2\sigma'\Delta\vartheta - A_1\lambda\frac{d\Delta\vartheta}{dr}, \quad (3)$$

where: A_1 is the area of the polished surface of the pulley rim; λ is the thermal conductivity coefficient of the pulley rim.

Thus, the intensity of the heat flow serves as the main parameter both when heating the pulley rim material and when causing large thermal stresses (the temperature gradient is included in the equation), which should be controlled during braking [6].

For friction pairs of brakes operating in an aperiodic intermittent mode, it is necessary to determine the maximum value of Q at which maximum thermal stresses arise, causing cracking of the working surface of the brake pulley rim.

The total braking work is equal to the sum of the works of the translational and rotational moving parts of the system (for example, the brakes of a drilling winch).

$$W = \frac{G}{g} \cdot \frac{v^2}{2} + I_{dsh} \frac{\omega^2}{2},$$

where $G = G_s + G'N$ is the total weight acting on the hook; G_s is the weight of the hoisting system, turbodrill, and drill collar; $G'N$ is the weight of the drilling tool; v is the hook speed; I_{dsh} is the moment of inertia of the drawworks drum shaft; ω is the angular velocity of the drum; N is the number of stands lowered into the borehole [7-8].

Provided that during braking all the work of friction is converted into heat $W = > Q$, we will compose the heat balance equation

$$Q = \frac{m_p c \Delta \vartheta}{1 - \alpha_{hf}}$$

where m_p is the mass of the pulley rim taking into account the effective heat penetration thickness; α_{hf} is the heat flow distribution coefficient.

In the equation of the controlled object we take as the control parameter $b \frac{d\Delta \vartheta}{dr}$ taking into account (3) and (2), we obtain

$$\frac{d\Delta \vartheta}{d\tau} = \frac{A_1 \lambda}{cm} \cdot \frac{d\Delta V}{dr} = b \frac{d\Delta V}{dr} = U(r, z, \tau). \tag{4}$$

where: $b = A_1/l_v$.

Let's compose a differential equation of heat conduction for a cylindrical pulley.

$$\frac{d\Delta \vartheta}{d\tau} = a \left(\frac{1}{r} \cdot \frac{d\Delta \vartheta}{dr} + \frac{d^2 \Delta \vartheta}{dr^2} + \frac{d^2 \Delta \vartheta}{dz_m^2} \right). \tag{5}$$

with initial

$$DV(r, z, 0) = 0 \tag{6}$$

and boundary at ($r = R$)

$$A_1 \lambda \frac{d\Delta \vartheta}{dr} + A_2 \sigma' \Delta \vartheta = Q \tag{7}$$

conditions that are necessary and sufficient for its solution using a generalized parameter.

$$x = \frac{r^2 + z_m^2}{a\tau}$$

Introducing a generalized parameter $x = \frac{r^2 + z_m^2}{a\tau}$ equation (5) will be reduced to the form

$$4x \frac{d^2 \Delta \vartheta}{dx^2} + (6 + x) \frac{d\Delta \vartheta}{dx} = 0. \tag{8}$$

The solution to equation (8) can be represented as:

$$\frac{d\Delta \vartheta}{dx} + C_1 e^{-\frac{1}{4}x} x^{-\frac{3}{2}}; \Delta \vartheta = C_1 \int x^{-\frac{1}{4}} x^{-\frac{3}{2}} dx + C_2 \tag{9}$$

Thus, the generalized parameter allows us to limit ourselves to only two (6) and (7) instead of four boundary conditions and one initial condition, which $\Delta \vartheta(\infty) = 0$

$$C_1 \left[\frac{2R^3}{R^2 + z_m^2} \cdot \frac{e^{-\frac{1}{4}x_0}}{\sqrt{x_0}} - \frac{KB_i}{x_0} \int e^{-\frac{x}{4} - \frac{3}{2}x} dx \right] = \frac{QR}{A_1 \lambda}, \quad (10)$$

where $KB_i = A_2 s' R / A_1 \lambda$; K - coefficient of volumetric deformation; B_i - Biot criterion; R - nominal radius of the pulley rim; z_m - generalized parameter.

From the first condition (10) we determine

$$C_2 = C_1 \left(\int e^{-\frac{x}{4} - \frac{3}{2}x} dx \right)_{x=\infty}, \quad (11)$$

and substituting the second, we get

$$C_1 = \frac{\frac{QR}{A_1 \lambda}}{\frac{2R^2}{R^2 + z_m^2} \cdot \frac{e^{-x_0/4}}{\sqrt{x_0}} - \frac{KB_i}{x_0} \int e^{-x/4} x^{-3/2} dx}, \quad (12)$$

where $x_0 = \frac{R^2 + z_m^2}{dz}$.

The improper integral in expressions (9, 10, and 12) can be represented as

$$\int_{x_0}^{\infty} e^{-\frac{x}{4} - \frac{3}{2}x} dx = \frac{2e^{-\frac{x}{4}}}{\sqrt{x}} - \sqrt{\pi} \frac{2}{\sqrt{\pi}} \int_{\frac{\sqrt{x}}{2}}^{\infty} e^{-z^2} dz = \frac{2e^{-\frac{x}{4}}}{\sqrt{x}} \cdot \sqrt{\pi} \operatorname{erfc}\left(\frac{\sqrt{x}}{2}\right), \quad (13)$$

where the latter is the Gauss integral, which we write as

$$\frac{2}{\sqrt{\pi}} \int_{\frac{\sqrt{x}}{2}}^{\infty} e^{-z^2} dz = \operatorname{erfc}\left(\frac{\sqrt{x}}{2}\right). \quad (14)$$

For large values of the argument, we expand function (14) into a series and, with sufficient accuracy, restrict ourselves to two of its terms.

$$\operatorname{erfc}\left(\frac{\sqrt{x_0}}{2}\right) = 2 \frac{e^{-x_0/4}}{\sqrt{\pi x_0}} \left(1 - \frac{2}{x_0}\right). \quad (15)$$

Taking into account (15) in expression (13) and the fact that $R^2 \gg z_m^2$, we simplify the value of C_1 :

$$C_1 = \frac{QR}{2A_1\lambda} \cdot \frac{\sqrt{x_0}e^{-x_0/4}}{\left(\frac{R^2}{R^2 + z_m^2} + \frac{2KBi}{x_0}\right)} \cong \frac{QR}{2A_1\lambda} \cdot \frac{x_0\sqrt{x_0}e^{-x_0/4}}{(x_0 + 2KBi)}. \quad (16)$$

where $K=A_2/A_1$.

Substituting (16) into (9) after some transformations, we obtain the equation for the control function [9]

$$\frac{d\Delta\mathcal{G}}{d\tau} = \frac{A_1\lambda}{cm} \cdot \frac{d\Delta\mathcal{G}}{dr} = \frac{QrR}{cm\left(R^2 + z_m^2 + 2KBi\alpha\tau\right)}. \quad (17)$$

Thus, using the method of thermal displacement potentials, thermal stresses in a heated cylindrical body were determined for the quasi-temperature case. It was established that cracking of the rim material occurs under the influence of a maximum stress difference, which has the form

$$\sigma_g = \sigma_{2r} - \sigma_{OU} = K^* \left(\frac{\Delta\mathcal{G}}{x} - \frac{d\Delta\mathcal{G}}{dx} \right) \quad (18)$$

Next, we will solve a variational problem that allows us to determine a mathematical algorithm for the control function $U(t)$, in which the control object is transferred from the initial position (at $\tau = 0$, $U = U_0$) to some final position (at $\tau = \tau_r$, $U = U_r$) and yields a minimum of the integral.

$$J_1 = \int_0^{\tau_r} U^2(t) d\tau. \quad (19)$$

For a limited value of maximum thermal stresses (isoparametric problem)

$$J_2 = \frac{K^*}{\tau_m} \int_0^{\tau_r} \left(\frac{\Delta\mathcal{G}}{x} - \frac{d\Delta\mathcal{G}}{dx} \right) d\tau \leq \sigma_g. \quad (20)$$

$$\frac{d\Delta\mathcal{G}}{dx} = \frac{d\Delta\mathcal{G}}{d\tau} \cdot \frac{d\tau}{dx} = -\frac{\tau}{x} \cdot \frac{d\Delta\mathcal{G}}{d\tau} = -\frac{\tau}{x} U(\tau), \quad (21)$$

taking into account the new coordinates on the pulley rim, we write

$$\left. \begin{aligned} \frac{d\Delta\mathcal{G}}{d\tau} &= U(\tau) = f_1; \\ \frac{dx_1}{d\tau} &= \frac{K^*}{\tau_m x} (\Delta\mathcal{G} + \tau U) = f_2; \\ \frac{dx_0}{d\tau} &= U^2(\tau) = f_3. \end{aligned} \right\} \quad (22)$$

Since all equations (22) are linear, we compose the Hamiltonian functions [10-11]

$$H = \sum \psi_1 f_1 = \psi_1 j(\tau) + \psi_2 \frac{K^*}{\tau m^x} (\Delta g + \tau U) + \psi_3 g^2(\tau) \quad (23)$$

and find its extreme value:

$$\frac{dH}{dU} = \psi_1 + \frac{\psi_2 K^* \tau}{\tau m^x} + 2\psi_3 U(\tau) = 0; \quad U(\tau) = - \left(\frac{\psi_1}{2\psi_3} + \frac{\psi_2 K^* \tau}{2\psi_3 \tau m^x} \right), \quad (24)$$

Since all equations (22) are linear, we compose the Hamiltonian functions

$$\frac{d\psi_i}{d\tau} = \sum_{i=1}^3 \frac{df_i}{dx_i} \psi_i. \quad (25)$$

We can represent its solution as follows:

$$\psi_1 = C_1' - \frac{\psi_2 Q K^* \tau^2}{2\tau m (r^2 + z_m^2)}; \quad \psi_2 = C_2' = const; \quad \psi_3 = C_3' = const. \quad (26)$$

Taking into account (26) and (24), we obtain

$$U(\tau) = - \frac{C_1'}{2\psi_3} + \frac{\psi_2 Q K^*}{2\psi_3 (r^2 + z_m^2)} \cdot \frac{\tau^2}{2\tau m} \cdot \frac{\psi_3 Q K^* \tau^2}{2\psi_3 (r^2 + z_m^2) \tau m} = C_1 - C_2 \tau^2 \quad (27)$$

where C_1 and C_2 are constants determined from expression (17) at $\tau = 0$; $U = U_0$, and accordingly $Q = Q_0$.

$$U_0 = \frac{Q_0 r R}{cm (R^2 + z_m^2)}. \quad (28)$$

From the integral relation (20), taking into account (27), we determine

$$\sigma_g \geq - \frac{K^* Q}{\tau m (r^2 + z_m^2)} \int_0^\tau \left[\int_0^\tau U(\tau) d\tau + \tau U(\tau) \right] \tau d\tau = \frac{2K^* a \tau_m^3}{3\tau m (r^2 + z_m^2)} \left(C_1 + C_2 \frac{2}{3} \tau_m^2 \right) \quad (29)$$

From relations (27), (28), (29) we find the values of the constants C_1 and C_2 :

$$C_1 = \frac{Q_0 r R}{cm (R^2 + z_m^2)}; \quad C_2 = \frac{3}{2\tau_m^2} \left[\frac{Q r R}{cm (R^2 + z_m^2)} - \frac{3\sigma_g x_m}{2K^* \tau_m} \right], \quad (30)$$

$$\text{where } x_m = \frac{r^2 + z_m^2}{dt_m}.$$

Taking into account (27) and (28), we obtain

$$U(\tau) = \frac{Q_0 r R}{cm(R^2 + z_m^2)} \left[1 - \frac{3}{2} \left(\frac{\tau}{\tau_m} \right)^2 + \frac{g}{4} \left(\frac{\tau}{\tau_m} \right)^2 \frac{\sigma_g cm (R^2 + z_m^2)^x}{Q_0 K^* r R \tau_m} \right]. \quad (31)$$

Using equation (31), we determine the optimal heat dissipation during heating of brake pairs and the temperature limits of applicability of alloys for the manufacture of pulleys.

To this end, we substitute the value of the control function, i.e., the heating rate $U(\tau)$ from (31), into (19) to find the integral minimum. The function that ensures the integral minimum allows us to determine the optimal heat dissipation during heating of friction pairs, thereby reducing the heating rate of the materials used and reducing their thermal stress [12].

For ease of calculation, we will denote

$$D = \frac{Q_0 r R}{cm(R^2 + z_m^2)}; \quad B = \frac{\sigma_g cm (R^2 + z_m^2)^x}{Q_0 K^* + r R \tau_m}; \quad U(\tau) = D \left[1 - \frac{\tau^2}{\tau_m^2} \left(\frac{3}{2} + \frac{3}{4} B \right) \right]$$

$$A = \frac{3}{2\tau_m^2} \left(1 + \frac{3}{2} B \right).$$

Then

$$U(\tau) = (1 - A\tau^2)DV.$$

Let's substitute the value of $U(\tau)$ into (19):

$$J_1 = \int_0^{\tau} U^2(\tau) d\tau = \int_0^{\tau} (1 - A\tau^2)^2 D^2 d\tau = D^2 \left(\tau - 2A \frac{\tau^3}{3} + A^2 \frac{\tau^5}{5} \right). \quad (32)$$

Thus, mathematical control of the energy load of the friction pairs of a drilling winch's band-and-shoe brake has been achieved.

The value of the maximum permissible thermal direction was selected for each material from the reference book [13-14] or calculated using the formula $\sigma_{\tau} = \beta E \Delta \vartheta$

.

It is known that temperature changes on the brake pulley surface are cyclical and depend on the number of descents and ascents. At the beginning of the descent, the increment is insignificant. The temperature increase increases with increasing braking time. However, thermal equilibrium is not achieved due to the simultaneous increase in heat transfer and the increase in braking work during the descent.

The resulting control function (31) optimizes heat dissipation during heating of the brake pairs and identifies the temperature limits of applicability of various alloys from which the pulleys are made.

Let's give an example of calculating the minimum of the integral J_1 using formula (32). Initial data:

$Q_0 = 60 \cdot 10^5$ J, $R = 0,725$ m, $C = 0,44 \cdot 10^3$ J/kg \cdot °C, $m = 140$ kg, $r = 0,695$ m, $a = 10,5 \cdot 10^{-6}$ m²/s, $\beta = 1,25 \cdot 10^{-5}$ °C⁻¹, $E = 2,185 \cdot 10^7$ Pa, $\tau_r = 10$ s, $\tau = (0 \dots 10)$ s, $z_r = 0,230$ m, $\mu = 0,35$.

Let's calculate

$$K^* = \frac{4E\beta}{1-\mu} = \frac{4 \cdot 2,185 \cdot 10^7 \cdot 1,25 \cdot 10^{-5}}{1-0,35} = 168 \cdot 10^5 ;$$

$$x_m = \frac{r^2 + z_m^2}{at_m} = \frac{0,695^2 + 0,230^2}{10,5 \cdot 10^{-6} \cdot 10} = 5 \cdot 10^3$$

The main stages of the development of tribology, like the development of other sciences, are associated with major scientific inventions and discoveries, which are patented, so we assume:

Previously unknown patterns have been established in the occurrence and development of microcracks on the working surfaces of metal friction elements in areas where mechanical stress concentrators are present and in areas where thermal stresses occur during electrothermomechanical friction of macroprotrusions of metal-polymer pairs. These patterns consist in the fact that under the influence of mechanical, electrical and thermal fields of a pulsed nature, the destruction of thin films of secondary structures occurs in the surface layer formed from the contact spots of the macroprotrusions, and their surfaces are subject to mechanical and thermal distortion under current stress waves, which include constant mechanical and residual stresses, and electrical and thermal currents of varying intensity act, weakening the surface layer; "At the same time, variable longitudinal and transverse temperature gradients are formed on the surface and subsurface layers of the metal friction element along its length and thickness, facilitating aperiodic cyclic processes of 'expansion (heating) – compression (cooling)', causing a disruption of the thermal balance and, as a consequence, the formation of a network of microcracks in the form of a fractal structure consisting of polygons, i.e., fused triangles with different areas of thermal stress concentration and minimal cross-sectional thicknesses of the subsurface layer of the metal friction element."

Computational experiment.

Research and study of friction processes in brake assembly friction units should be conducted not only from the perspective of the dynamic and thermal loading of their friction pairs, but also by considering each of the materials from which the metal friction element and linings are made. Let's consider the selection of materials based on mechanical properties for brake pulleys in drilling winches.

The calculation examples used the mechanical characteristics of steels recommended in [15]: 35KhNL (standard) with a tensile strength of $s_p = 72 \cdot 10^7$ Pa;

25KhGSL - $s_p = 74 \cdot 10^7$ Pa; 15KhNMF - $s_p = 85 \cdot 10^7$ Pa; 20Kh2MF - $s_p = 100 \cdot 10^7$ Pa.

$$D = \frac{60 \cdot 10^5 \cdot 0,695 \cdot 0,725}{0,44 \cdot 10^3 \cdot 14 \left(0,725^2 + 0,230^2 \right)} = 848 ;$$

$$B = \frac{70 \cdot 10^7 \cdot 0,44 \cdot 10^3 \cdot 14 \left(0,725^2 + 0,230^2 \right) \cdot 5 \cdot 10^3}{60 \cdot 10^5 \cdot 168 \cdot 10^5 \cdot 0,695 \cdot 0,725 \cdot 10} = 24,55 ;$$

$$A = \frac{3}{2 \cdot 10^2} \left(1 + \frac{3}{2} \cdot 24,5 \right) = 0,56 ;$$

$$J_1 = 848^2 \left(10 \cdot 2 \cdot 0,56 \frac{10^3}{3} + 0,56^2 \frac{10^5}{5} \right) = 41,6 \cdot 10^8 .$$

For material 25KhGSL

$$B = \frac{75 \cdot 10^7 \cdot 0,44 \cdot 10^3 \cdot 14 \left(0,725^2 + 0,230^2 \right) \cdot 5 \cdot 10^3}{60 \cdot 10^5 \cdot 168 \cdot 10^5 \cdot 0,695 \cdot 0,725 \cdot 10} = 26,3 ;$$

$$A = \frac{3}{2 \cdot 10^2} \left(1 + \frac{3}{2} \cdot 26,3 \right) = 0,65 ;$$

$$J_{1,2} = 848^2 \left(10 - 2 \cdot 0,65 \frac{10^3}{3} + 0,65^2 \frac{10^5}{5} \right) = 48,2 \cdot 10^8 ;$$

For material 15KhNMF

$$B = \frac{85 \cdot 10^7 \cdot 0,44 \cdot 10^3 \cdot 14 \left(0,725^2 + 0,230^2 \right) \cdot 5 \cdot 10^3}{60 \cdot 10^5 \cdot 168 \cdot 10^5 \cdot 0,695 \cdot 0,725 \cdot 10} = 29,8 ;$$

$$A = \frac{3}{2 \cdot 10^2} \left(1 + \frac{3}{2} \cdot 29,8 \right) = 0,67 ;$$

$$J_{1,3} = 848^2 \left(10 - 2 \cdot 0,67 \frac{10^3}{3} + 0,67^2 \frac{10^5}{5} \right) = 61,5 \cdot 10^8 ;$$

For material 20Kh2MF material

$$B = \frac{100 \cdot 10^7 \cdot 0,44 \cdot 10^3 \cdot 14 \left(0,725^2 + 0,230^2 \right) \cdot 5 \cdot 10^3}{60 \cdot 10^5 \cdot 168 \cdot 10^5 \cdot 0,695 \cdot 0,725 \cdot 10} = 35,0 ;$$

$$A = \frac{3}{2 \cdot 10^2} \left(1 + \frac{3}{2} \cdot 35 \right) = 0,80 ;$$

$$J_{1,4} = 848^2 \left(10 - 2 \cdot 0,80 \frac{10^3}{3} + 0,80^2 \frac{10^5}{5} \right) = 88,4 \cdot 10^8$$

The minimum J_1 function is achieved with grade 35KhNL material, which exhibits optimal heat dissipation during friction pair heating. This reduces the heating rate and the generation of thermal stress on the brake pulley working surface.

Discussion of results.

Theoretical research and computational experiments devoted to optimal control of thermal processes in friction pairs during frictional interaction in braking devices have allowed us to establish and propose:

- thermal process control is accomplished mathematically using the heat conduction equation with initial and boundary conditions, using a control function, as well as the method of thermoelastic potentials, moving local heat sources in the brake friction pairs;
- selection of material based on the mechanical properties of brake pulleys for drilling winches using a computational experiment\$
- based on the completed research, a discovery formula was formulated: "Unknown patterns of occurrence and development of microcracks on the working surfaces of metal friction brake elements have been established".

Conclusions. Thus, the problem of controlling the energy load of the friction pairs of a band-shoe brake has been solved using mathematical tools and computational experiments.

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КЕРУВАННЯ ЕНЕРГОНАВАНТАЖЕНІСТЮ ПАР ТЕРТЯ ГАЛЬМ

Теоретичні дослідження та обчислювальний експеримент присвячені оптимальному керуванню тепловими процесами пар тертя при фрикційній взаємодії у гальмових пристроях дозволили встановити та запропонувати наступне. Керування тепловими процесами здійснено математичним шляхом на основі рівняння теплопровідності з початковими та граничними умовами із залученням керуючої функції, а також методу термопружних потенціалів, переміщуючих локальних джерел теплоти в парах тертя гальма. Підбір матеріалу за механічними властивостями гальмових шківів бурових лебідок за допомогою обчислювального експерименту. На підставі виконаних досліджень сформульовано формулу відкриття «Встановлено невідомі закономірності виникнення та розвитку мікротріщин на робочих поверхнях металевих фрикційних елементів гальм».

Ключові слова: керування, енергонавантаженість, гальмові пристрої, пара тертя, еквівалентні напруження.

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