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PARAMETRIC EFFECT OF STEADY AND UNSTEADY FRICTION ON THE ORIGIN AND INITIAL PROPAGATION OF THE SHOCK PULSE

The phenomenon of unsteady flow is widespread in engineering. It is known to generate a shock pulse (water hammer). The nature of this phenomenon is nonlinear and depends on several factors: steady friction, unsteady friction, convection of velocity and pressure fields, bulk viscosity, and the conditions under which the shock pulse occurs. Using models in modern software requires an accurate understanding of the physics of the generation and primary propagation of a shock pulse—a nonlinear process that, under certain conditions, allows for the existence of more than one fluid flow mode. Therefore, a parametric study of previously obtained analytical solutions is of practical interest. The results presented in this paper highlight the importance of parameters corresponding to steady and unsteady friction. While steady friction affects the size of the shock pulse region (the size decreases with increasing friction), unsteady friction has a completely different effect. As the parameter responsible for unsteady friction increases, the possibility of the existence of two shock pulses—one weaker and one stronger—emerges. In this case, hydraulic concepts based on the Bernoulli equation prove inapplicable: higher pressure values correspond to higher shock pulse propagation velocities. This is entirely logical: the greater the non-stationarity (and the shock pulse propagation velocity), the greater the pressure surges.

Keywords: *unsteady flow, water hammer, steady and unsteady friction against the pipe wall, parametric effect*

Introduction. The phenomenon of water hammer can occur in various fluid media as unsteady flows (called also as transients) in pipelines made of various materials. For example, the differences between unsteady flows in viscoelastic and elastic media are expressed in the consideration of a 2D model, in which elastic behavior is assumed for the pipe material [1].

In the viscoelastic model, the pressure and velocity profiles decay more rapidly, indicating the significant influence of viscosity on the shock pulse propagation process [1]. An important feature is that the velocity profiles obtained by the viscoelastic model are generally flatter than those obtained by the elastic model. Consequently, viscosity smooths out the velocity and pressure gradients, making them flatter.

Not only water but also various non-Newtonian fluids flow through industrial pipelines [2]. Such fluids include liquid construction materials (slurries), sewage, pastes, and suspensions [2]. These fluids are viscoplastic. A mathematical model of viscoplastic water hammer is presented in [3].

Currently, humanity is susceptible to a variety of diseases, among which cardiovascular diseases are the most common. This is especially true for Ukraine, as the country affected by the Chernobyl nuclear power plant disaster in 1986. Ruptured blood vessels lead to sudden death. The cause of ruptured blood vessels is a blood hammer (blood stroke). Therefore, studying blood hammer from the perspective of a non-Newtonian fluid (blood is a non-Newtonian fluid) is also an important practical task. The study [4] is devoted to the non-Newtonian properties of blood on blood hammer. According to these studies, these properties significantly influence the velocity and shear stress. Also noteworthy is the study [5], which presents an analytical model of blood hammer.

Of interest from the perspective of the diversity of hydraulic pulse is the paper [6], which examines water hammer in a polymer pipe based on a two-dimensional (2-D Kelvin - Vogt) model. Since modern pipes are primarily made of polymers, the research [6] is important.

During a water hammer, not only does pressure increase, but it also drops sharply, to near-zero value. Under these conditions, hydrodynamic cavitation occurs, and the liquid, after undergoing such a water hammer, contains gas bubbles and is now a two-phase medium. A detailed review of this phenomenon can be found in the paper [7]. A review of various water hammer models is provided in the work [8].

After conducting a rather diverse review of the shock pulse propagation process in various fluid media and structural materials, the impact of this diversity becomes clear. Let's move on to a more specific class of hydraulic water hammer problems in Newtonian fluids, namely, the parametric effect of steady and unsteady friction, taking into account the convection of velocity and pressure fields, on the origin and propagation of shock pulse.

Problem state. Historically, the phenomenon of water hammer was first studied for water flow in irrigation systems [9] and much later in the construction of multi-story buildings [10]. Since in these cases the length of the pipeline is quite large (many tens and hundreds of meters), the convective components of the velocity and pressure fields can be ignored. In these cases, the linear theory of water hammer was used. The rapid development of industry in the 20th century led to a revision of the existing theory of this phenomenon. Thus, the phenomenon of water hammer has also been observed in very short flexible pipes in aviation hydraulic sysytems, the working pressure in which is very high - 200 atm. In this case, pressure surges reach 75% of the working pressure [11]. In other words, in various technologies, nonlinear effects should be taken into account, that is, the convection of velocity and pressure fields [12,13,14]. In the work [15], the theory of water hammer was further developed.

Since the phenomenon of water hammer is classified as a unsteady flow, it became necessary to develop models that account for the unsteady friction of fluid against the pipeline walls. Thus, in addition to the existing Weisbach -Darcy model of steady friction [16-17], new models for describing unsteady friction were elaborated. They are presented in the works of Brunone [18,19] and Vitkovsky [20,21], as well as in Pezzinga [22].

At the end of the last century, automated design and calculation systems for various physical processes have been developed, primarily for problems in fluid and gas mechanics. These systems allow for the numerical solution of complex fluid flow problems, including water hammer, at the engineering level. The accuracy of fluid flow calculations, which are described by a nonlinear model, largely depends on a correct (precise) description of the shock pulse formation phenomenon – its origin and initial propagation. The studies of this phenomenon are devoted to [12, 13, 14]. A more generalized model of water hammer is presented in [15]. The process is described by equations

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\lambda}{8\mathrm{R}}V\left|V\right|+kD\left[\frac{\partial V}{\partial t}+C_f\,sign\left(V\right)\left|\frac{\partial V}{\partial z}\right|\right]=0\,, \tag{1}$$

$$\frac{\partial p}{\partial t}+V\frac{\partial p}{\partial z}+a^2\rho_0\frac{\partial V}{\partial z}=0\,. \tag{2}$$

General and particular solutions of the system (1)-(2) are obtained in the work [15]. Self-similar dimensionless form for (1)-(2), necessary for further consideration, is expressed by the following two equations (see details in [15]):

$$-\frac{d\bar V}{d\bar\eta}+\bar V\frac{d\bar V}{d\bar\eta}+\frac{d\bar p}{d\bar\eta}+\mathrm{DW}\bar V\left|\bar V\right|+\mathrm{Br}\left[-\frac{d\bar V}{d\bar\eta}+sign\left(\bar V\right)\left|\frac{d\bar V}{d\bar\eta}\right|\right]=0\,, \tag{3}$$

$$\frac{d\bar V}{d\bar\eta}-\frac{d\bar p}{d\bar\eta}+\bar V\frac{d\bar p}{d\bar\eta}=0\,. \tag{4}$$

In (3) DW and Br are Darcy-Weisbach and Brounone numbers respectively (see their definition in [14,15]).

If we take into account only unsteady friction, without taking into account pressure convection, then the general self-similar solution (3)-(4) has the form [15]:

$$\bar\eta\left(\bar V\right)=\frac{1}{\mathrm{DW}}\left[\frac{2\mathrm{Br}}{\bar V}-\ln\left|\bar V\right|\right]+\mathrm{C}_1\,. \tag{5}$$

If, on the contrary, we do not take into account non-stationary friction, but only the convection of the velocity and pressure fields, then we obtain the following solution [15]:

$$\bar\eta\left(\bar V\right)=\pm\frac{1}{\mathrm{DW}}\left[\ln\left|\bar V-1\right|-2\ln\left|\bar V\right|\right]+\mathrm{C}_1\,. \tag{6}$$

In solution (4), the plus and minus signs correspond to the forward and backward shock pulses.

Finally, if we take into account both unsteady friction and pressure field convection, we obtain the following self-similar solution [15]:

$$\bar{\eta}\left(\bar{V}\right)=\frac{1}{DW}\left[\ln\left|\bar{V}-1\right|-2\ln\left|\bar{V}\right|-\frac{1}{\bar{V}}\left(2Br-1\right)\right]+C_1. \tag{7}$$

In [15] single dependencies were obtained using formulas (5)–(7). Therefore, there is no study of the parametric dependence of the shock pulse formation process.

Problem formulation. Conduct research into the parametric effect of steady and unsteady friction on the origin and initial propagation (i.e. formation) of a shock pulse.

The purpose of the work. Obtain graphical dependencies of the shape of the shock pulse on the values of the parameters corresponding to different types of friction and analyze these dependencies.

Explicit dependencies for self-similar solutions corresponding to forward and backward shock pulses. First, let's note the qualitative difference in the physics of forward and backward shock pulses. From equation (3), it follows that the backward shock pulse, for which the velocity derivative of the self-similar coordinate is negative, is not affected by unsteady friction: the expression in square brackets, preceded by the Brunone number, vanishes. Therefore, solutions (5) and (7) essentially correspond to forward shock pulses. The backward shock impulse does not contain the number Br.

To obtain explicit relationships from general solutions (5)-(7), one physical condition should be used. This is the condition of maximum velocity at the crest of the shock pulse, that is, at $\bar{\eta}=0$:

$$\bar{V}\left(\bar{\eta}=0\right)=\bar{V}_{\max}. \tag{8}$$

If we apply condition (8) to solutions (5) -- (7), we obtain the following explicit expressions:

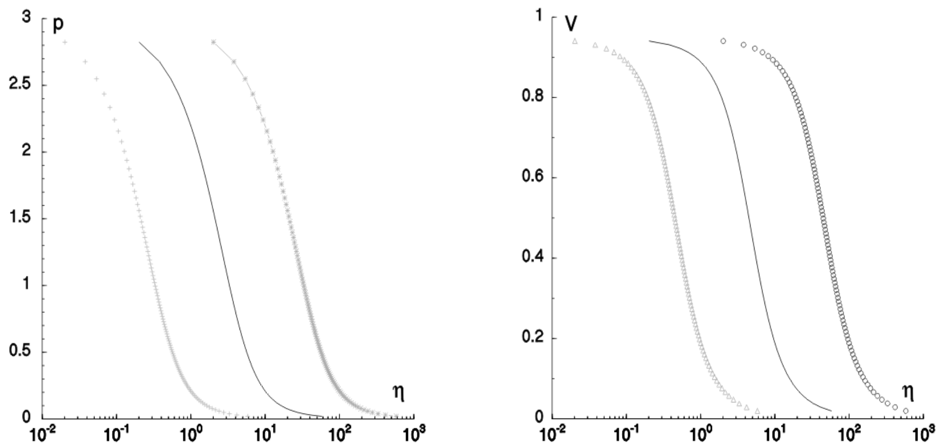
$$\bar{\eta}\left(\bar{V}\right)=\frac{1}{\mathrm{DW}}\left(\left[\frac{2\mathrm{Br}}{\bar{V}}-\ln\left|\bar{V}\right|\right]-\left[\frac{2\mathrm{Br}}{\bar{V}_{\max}}-\ln\left|\bar{V}_{\max}\right|\right]\right), \tag{9}$$

$$\bar{\eta}\left(\bar{V}\right)=\frac{1}{\mathrm{DW}}\left(\pm\left[\ln\left|\bar{V}-1\right|-2\ln\left|\bar{V}\right|\right]\mp\left[\ln\left|\bar{V}_{\max}-1\right|-2\ln\left|\bar{V}_{\max}\right|\right]\right), \tag{10}$$

$$\bar{\eta}\left(\bar{V}\right)=\frac{1}{\mathrm{DW}}\left(\left[\ln\left|\bar{V}-1\right|-2\ln\left|\bar{V}\right|-\frac{2\mathrm{Br}-1}{\bar{V}}\right]-\left[\ln\left|\bar{V}_{\max}-1\right|-2\ln\left|\bar{V}_{\max}\right|-\frac{2\mathrm{Br}-1}{\bar{V}_{\max}}\right]\right). \tag{11}$$

Formulas (9)-(11) will be used to obtain graphical dependencies for viscosity effect study.

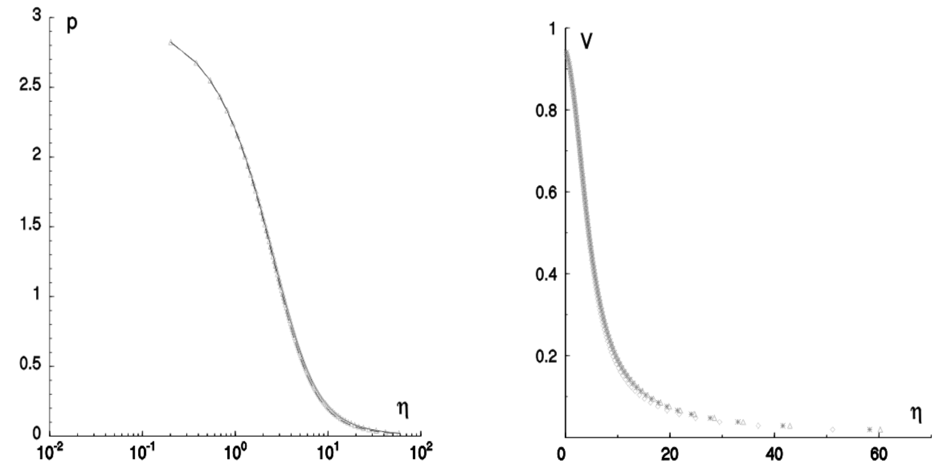
Parametric effect of non-steady and steady friction on the origin and initial propagation of a shock pulse. The effect of steady friction of a liquid against the pipe walls (the Darcy-Weisbach effect) is shown in Fig. 1. It is evident that, in dimensionless self-similar quantities, the Darcy-Weisbach friction significantly influences the area of the shock pulse region in the liquid. Moreover, it is entirely natural for this region to decrease in size with increasing friction on the pipeline walls. In other words, for large values of the DW parameter corresponds to a narrower area of passage of the shock pulse.



a) b)

Fig. 1. The effect of the parameter (steady friction) DW on the shape of the shock pulse: a) is the pressure profile for Darcy-Weisbach numbers of 10,1,0.1 (curves from left to right); b) is the velocity profile for Darcy-Weisbach numbers of 10,1,0.1 (curves from left to right).

Unlike steady friction, the effect of unsteady friction, that is, the parameter Br, is practically insignificant for small values. Therefore, the curves corresponding to the Brunone numbers 0.01, 0.03, 0.1 practically overlap each other. And this does not depend on the choice of the form of representation. Thus, whether in the semi-logarithmic coordinate system for pressure (see Fig. 2a), or in the conventional coordinate system for velocity (see Fig. 2b), the closeness of the calculation results is still obvious. But there is a common property of Figures 1-2. It consists in the fact that the values of the self-similar coordinate equal to zero are practically never achieved. What is the reason for this behavior of the solutions? The fact is that the system of equations (1)-(2) and (3)-(4) is nonlinear. Therefore, a nonlinear differential equation is obtained for the velocity, the solutions of which (9)-(11) do not yield values for the dimensionless self-similar coordinate equal to 0. From a physical point of view, this indicates the limitations of the model in the sense that no solution exists near the zero value of the self-similar coordinate. This absence may be due to the fact that bulk viscosity, which reveals itself quantitatively for small values of the self-similar coordinate, is not taken into account. The term with bulk viscosity transforms the differential equation into a singular one – with a small parameter at the highest (second) derivative. Therefore, to find a physically complete solution in the region of small values of the self-similar coordinate, it is necessary to separately consider the shock boundary layer – a thin region where bulk viscosity plays a significant role. However, this problem is not considered in this paper.

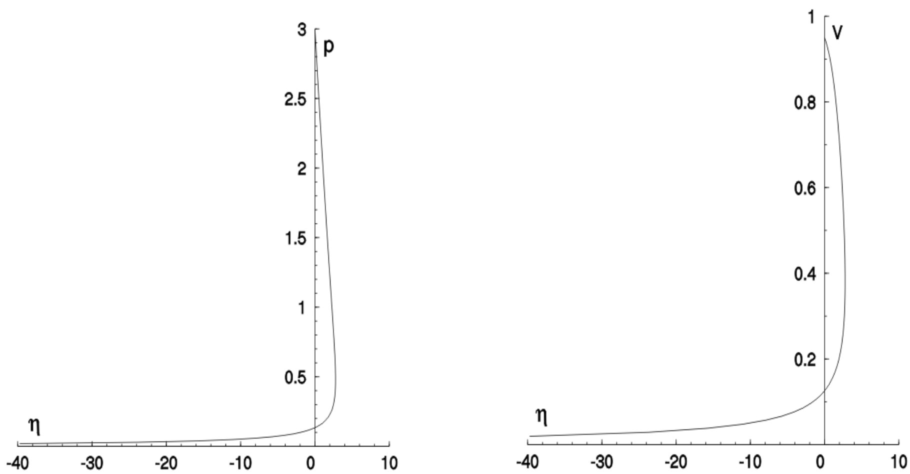


a) b)

Fig. 2. The effect of the parameter (unsteady friction) Br on the shape of the shock pulse: a)—pressure profile fort Br = 0.01, 0.03, 0.1; b)—velocity profile for Br = 0.01, 0.03, 0.1.

The effect of unsteady friction on the structure of the shock pulse begins to have a significant impact as the Bruno number increases, approaching 1. Thus, for a Bruno number of 1 (and a Darcy-Weisbach number of 1), the shock pulse region occupies only a few units of the dimensionless self-similar coordinate (see Fig. 3). And this is not the most important factor, as it is that at virtually every point in this region, two solutions are now possible—a weak solution and a stronger solution, in the sense of increased pressure (see Fig. 3a). The propagation velocities of the shock pulse can also have two different values. In this case, our concept, based on Bernoulli's law of decreasing pressure with increasing velocity, does not hold: lower values of the shock pulse propagation velocity correspond to lower pressure values. Although, from a common sense perspective, this is logical, since the weaker the flow unsteadiness, expressed by the propagation velocity of the shock pulse, the weaker the pulse itself and, consequently, the excess pressure within it. Another feature of the flow under consideration is the minimum value for the velocity and pressure functions in the shock pulse, since values of the self-similar variable that are less than zero do not correspond to the model.

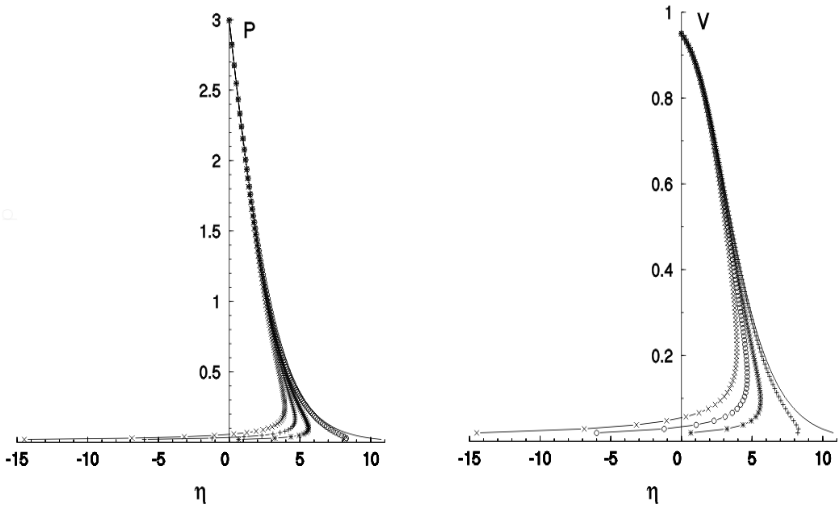
Based on what has just been said, a reasonable question arises: what is the minimum boundary value of the Brunone number at which the simultaneous existence of two shock pulses is possible? The answer to this question is presented in Figure 4. As it turns out, this value, accurate to three decimal places, is approximately $\mathrm{Br} \approx 0.525$. It is for these values of the parameter Br responsible for dimensionless friction that both the velocity and pressure curves begin to bend. Further, somewhere up to values $\mathrm{Br} \approx 0.6$, the possibility of the existence of two types of shock pulses—a weaker and a stronger one—is observed for virtually all values of the dimensionless self-similar variable η, right down to zero. As the Brunone number increases, the velocity and pressure curves cross the zero abscissa, thereby indicating the existence of minimum values of the shock pulse propagation velocity and the pressure within it—for a weak shock pulse.



a) b)

Fig. 3. Pressure and velocity in a shock pulse under strong unsteady friction. The graph lines correspond to **Br** = 1.

Conclusions. The theory of water hammer in conventional, i.e. Newtonian fluids, at the level of the hydraulic, i.e. one-dimensional, representation of the flow, operates with such concepts as steady (Darcy- Weisbach) friction of the fluid against the pipeline wall and unsteady (Brunone-Vitkovsky) friction of the same fluid against the pipeline wall. The presented studies of the parametric effect on the origin and initial propagation of the shock pulse indicate the physical diversity of the phenomenon. Thus, if the parameter DW of steady friction significantly influences the process over the entire range of its variation, then the parameter Br of unsteady friction reveal itself at values comparable to unity. From a quantitative point of view, an increase in the role of steady friction is reduced to a narrowing of the shock pulse region and vice versa. Moreover, the distribution of velocity and pressure are not determined for all values of the dimensionless coordinate – near zero they are undefined. And this can be explained by the fact that volume friction is not taken into account in the model (see [23]). The effect of unsteady friction is somewhat different. For values of the corresponding dimensionless number significantly less than unity (0.1 and less), the velocity and pressure curves practically overlap. Only at values comparable to unity does a sufficiently rapid quantitative and qualitative change in the shape of the shock pulse occur. This makes it possible for two shock pulses to exist simultaneously—a weaker one and a stronger one.



a) b)

Fig. 4. Profiles of dimensionless pressure a) and velocity b) for Bruno numbers Br = 0.52, 0.525, 0.6, 0.66, 0.75. The curves follow from right to left according to the listed Bruno numbers. The parameter DW (Darcy-Weisbach number) is always equal to 1.

As future research, the effect of bulk viscosity on the structure of shock pulse origin and primary propagation may be under the study.

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П.В. ЛУК'ЯНОВ

ПАРАМЕТРИЧНИЙ ВПЛИВ СТАЦІОНАРНОГО ТА НЕСТАЦІОНАРНОГО ТЕРТЯ НА ЗАРОДЖЕННЯ ТА ПЕРВИННЕ ПОШИРЕННЯ УДАРНОГО ІМПУЛЬСУ

Явище нестаціонарного течії поширене у техніці. Як відомо, воно призводить до формування ударного імпульсу (ударної хвилі). Природа цього явища є нелінійною і залежить від кількох факторів: стаціонарного тертя, нестаціонарного тертя, конвекції полів швидкості та тиску, об'ємної в'язкості, а також умов виникнення ударного імпульсу. Але, як би там не було, для використання моделей у сучасному програмному забезпеченні необхідно як найточніше представляти фізику зародження та первинного поширення ударного імпульсу – нелінійного процесу, що дозволяє, за певних умов, існування більш однієї моди течії рідини. Тому параметричне дослідження отриманих раніше аналітичних розв'язків становить практичний інтерес. Подані в даній роботі результати вказують на важливість параметрів, що відповідають стаціонарному та нестаціонарному тертю. Якщо стаціонарне тертя впливає на розміри області ударного імпульсу (розмір зменшується зі збільшенням тертя), то нестаціонарне тертя впливає зовсім інакше. При зростанні параметра, що відповідає за нестаціонарне тертя, виникає можливість існування двох ударних імпульсів – слабшого та сильнішого. При цьому гідравлічні уявлення, засновані на рівнянні Бернуллі, виявляються непридатними: сильнішим значенням тиску відповідають вищі значення швидкості поширення ударного імпульсу. І це цілком логічно: чим сильніша нестаціонарність (і швидкість поширення ударного імпульсу), тим сильніші стрибки тиску.

Ключові слова : нестаціонарна течія, гідравлічний удар, стаціонарне та нестаціонарне тертя рідини о стінку труби, параметричний вплив

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