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LOCAL EQUIVALENT STRESSES IN METAL FRICTION ELEMENTS OF BRAKING DEVICES

Theoretical research and a computational experiment applied to local equivalent stresses in metal friction elements of braking devices allowed us to establish and propose a flowchart for assessing the energy load of friction pairs, in which the assessment of equivalent stresses is presented last. A mathematical description of the relationship between local temperature zones and equivalent stresses in friction pairs. A computational and experimental study to determine equivalent stresses on the working surface of metal friction elements. A patent claim for "Method for the occurrence and prediction of a microcrack network on the working surface of metal elements at different rates" was submitted. Method: finite element modeling using the Ansys Workbench program for the stress state of metal friction elements.

Key words: *braking device, friction pair, metal element, heat exchange processes, stress*

Introduction. The operating conditions of friction pair materials in brake assemblies within tribosystems, i.e., under conditions of electrothermomechanical friction and wear, are unique in that the surface layers of the microprotrusion contact patches experience a variety of energetic effects, living in a complex stress-strain state. Dynamic and thermal loads cause high surface thermal stresses in the metal friction element, leading to the formation of a network of microcracks that contribute to its intense wear and failure.

Analysis of literary sources and the state of the problem. The study of the stress-strain state of an elastically rotating solid and annular disk is the subject of [1]. A plane stress state is approximately realized in this disk. Determining the stress and strain components was reduced to solving a system of two differential and two linear equations. The numerical solution, by analogy with the analytical one, was reduced to dimensionless form. Subsequently, the patterns of change in the components of the stress, strain, and displacement tensor from the radial coordinate of the disk were established. The results of the analytical solution were compared with the numerical solution obtained by the finite element method in the ANSYS Mechanical software package, which amounted to 6.5%.

The study of the physical patterns of change in the structural-phase and stress-strain state of the surface layer of component materials during friction, the accumulation and generalization of the results of experimental research and operating experience of tribosystems of the type and purpose made it possible to determine the physical foundations of the structural modification of tribosystem materials is the subject of [2]. In [3], a clear picture of the stress distribution was obtained for each design case, which allows one to identify dangerous zones of the aircraft brake wheel
design in terms of strength. Similar methods were used to analyze the stress-strain state of other brake wheel components (brake housing, cylinder block, wheel drum). Based

on the stress state analysis, measures were taken to strengthen the structure already at the design stage. In [4], a discrete model was proposed for studying the thermoelastic frictional interaction of an elastic wavy surface with a perfectly rigid smooth surface. The mathematical model of thermoelastic frictional contact is based on the instantaneous relief of the wavy surface, described by a vector $\overline{H}$, the terms of which are: the initial undeformed relief, separately taken mechanical and thermoelastic displacement, and wear of surface microprotrusions. Following this, the thermal and contact compliance matrices of the tribo-coupling microprotrusions were determined, taking into account various load-change laws acting on their contact patches [5]. However, the energy characteristics of the tribo-coupling microprotrusion contacts and their electrical and thermal resistance were not considered. It follows from the above that friction pairs require forced and compulsory cooling.

The aim of the research is to analyze a generalized method for assessing the equivalent stresses of metal friction elements of braking devices.

Determination of thermal stresses during heating and cooling of metal friction elements. The assessment of thermal stresses arising in brake pulleys under high surface temperature gradients, as well as under forced air cooling (residual stresses) [6], is important for the selection of their materials and optimal design parameters.

Fig. 1 shows a block diagram of the energy density of friction pairs of braking devices, in which attention is paid to their stresses. Fig. 2 is devoted to the connection heat release equal to Q. The one-dimensional heat equation in this case has the form

$$\frac{d\Delta\vartheta}{d\tau} = \alpha \frac{d^2\Delta\vartheta}{dx^2}. \qquad (1)$$

where: $d\vartheta/dt$ is the change in surface temperature over time; dx is the coordinate; a is the thermal diffusivity coefficient.

Equation (1) satisfies the following initial and boundary conditions

$$\left.\begin{aligned} &A_1\lambda\frac{d\Delta\vartheta}{dz} - A_2\sigma'\Delta\vartheta + Q = npu \quad x = 0. \\ &\Delta\vartheta(x,0) = 0 \end{aligned}\right\}. \qquad (2)$$

where: λ is the thermal conductivity coefficient; A_1, A_2 are the areas of the polished and matte surface of the metal element; $\Delta\vartheta$ is the temperature increase; σ' is the heat transfer coefficient [7].

Conditions (2) are necessary and sufficient for solving the differential equation (1) using the generalized parameter dz [8].

We move on to dimensionless coordinates, and also designating we have

$$\Delta\vartheta = \frac{\vartheta(x,\tau) - \vartheta_{aver}}{\vartheta_{aver}}; \quad \overline{x} = \frac{x}{b}, \; \overline{\tau} = \frac{a\tau}{b^2} = F_O; \quad K = \frac{A_2}{A_1};$$

$$\overline{Q} = Qb / A_1\lambda\vartheta_{aver} \qquad Bi = \sigma b / A_1,$$

where: ϑ_{aver} – ambient temperature; Fo, Bi – Fourier and Bio criterion.

We write equations (1) and (2) in the form and obtain

$$\frac{d\Delta\overline{\vartheta}}{d\overline{\tau}} = \frac{d^2\Delta\overline{\vartheta}}{d\overline{x}^2}; \quad (3)$$

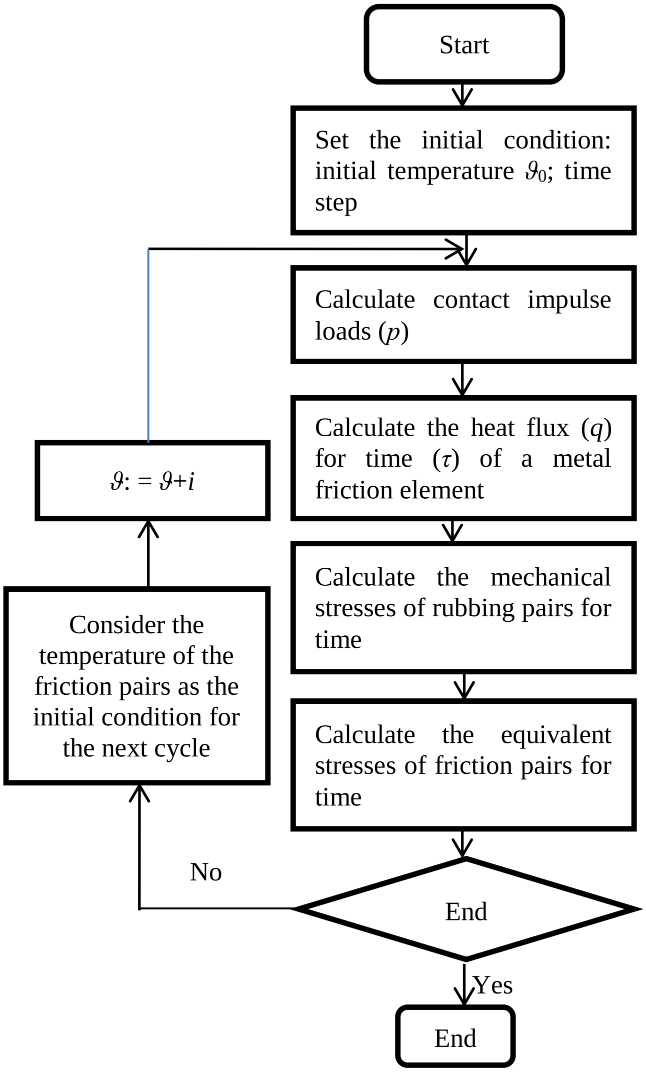

Fig. 1. Block diagram for assessing the energy load of friction pairs of braking devices

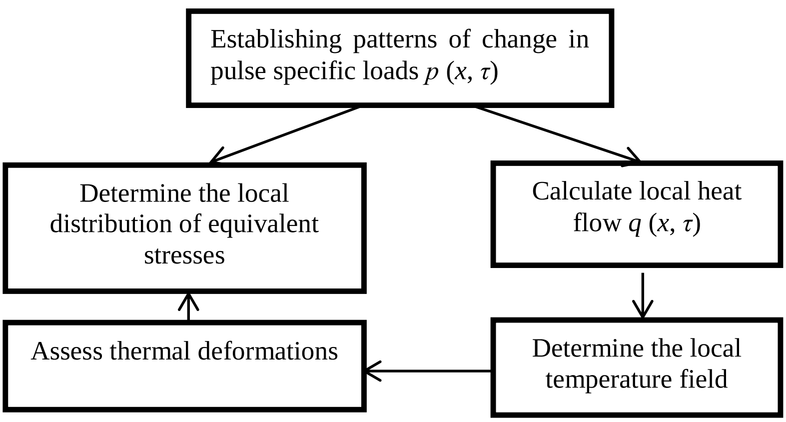

Fig. 2. Relationship between the temperature field and equivalent stresses of brake friction pairs

$$\left.\begin{array}{l}\dfrac{d\Delta\overline{\vartheta}}{d\overline{x}} = KB_i\Delta\overline{\vartheta} + Q \;\; at \;\; \overline{x} = 0(z = 0); \\ \Delta\overline{\vartheta}(x,0) = 0\end{array}\right\}. \qquad (4)$$

By introducing a generalized parameter $z = x^2 / a\tau = \overline{x}^2 / \overline{\tau}$, having represented the differential equation (12), we represent it as

$$4z\frac{d^2\Delta\overline{\vartheta}}{dz^2} + (2 + z)\frac{d\Delta\overline{\vartheta}}{dz} = 0. \qquad (5)$$

The solution to equation (5) is:

$$d\Delta\overline{\vartheta} = c_1 e^{-\frac{1}{4}z} z^{-\frac{1}{2}}; \; \frac{d\Delta\overline{\vartheta}}{d\overline{x}} = \frac{2\overline{x}}{\overline{\tau}} \cdot \frac{d\Delta\overline{\vartheta}}{dz} = \frac{2c_1}{\sqrt{\overline{\tau}}} e^{-\frac{1}{4}z};$$

$$\Delta\overline{V} = C_1 \int e^{-\frac{1}{4}z} z^{-\frac{1}{2}} dz + C_2. \qquad (6)$$

where: c_1 is the specific heat capacity of the metal element; C_1, C_2 are constant numerical values [9].

Solution (6) must satisfy conditions (4) at z = 0:

$$\frac{2a}{\sqrt{\overline{\tau}}} - KBi\left[\left(a\int e^{-\frac{1}{4}z} z^{-\frac{1}{2}} dz\right)_{z=0} + C_2\right] + \overline{Q} = 0,$$

at $z = \infty$ ΔV=0. (7)

The solution of equation (5) satisfies conditions (7):

$$\Delta\overline{\vartheta} = \frac{\overline{Q}}{KBi + \frac{1}{\sqrt{\pi\overline{\tau}}}}\left[1 - erf\left(\frac{\overline{x}}{2\sqrt{\overline{\tau}}}\right)\right] = \frac{Q}{KBi + \frac{1}{\sqrt{\pi\overline{\tau}}}} erf\left(\frac{\overline{x}}{2\sqrt{\overline{\tau}}}\right), \qquad (8)$$

where: $erf\left(\frac{\overline{x}}{2\sqrt{\overline{\tau}}}\right) = \frac{1}{\sqrt{\pi}}\int e^{-\frac{1}{4}z} z^{-\frac{1}{2}} dz = 4\frac{2}{\sqrt{\pi}}\int\limits_0^U e^{-U^2} dU$ - Gauss integral; U(t) is the control function.

The surface temperature of the metal friction element is determined from (8) at

$$\overline{x} = 0: \quad \Delta\vartheta_M(t) = Q \Big/ \left(KBi + \frac{1}{\sqrt{\pi\overline{\tau}}}\right). \qquad (9)$$

where: $\Delta\vartheta_M$ is the increment at the point under consideration [10].

From (9) we obtain the temperature of the steady-state difference in surface temperature $\overline{t} \to \infty$ at which $\Delta\overline{\vartheta} = Q / KBi$. (10)

During aperiodic, short-term intermittent braking, forced air cooling of the friction pair occurs to the initial temperature or higher, with the temperature increasing from braking to braking. The cooling time is typically several times longer than the braking time [11].

Let's determine the changes in temperature difference during cooling. Equation (10) must satisfy the following boundary conditions:

$$at\ \bar{x} = z = 0 \quad A_{t1}\lambda \frac{d\Delta\bar{\vartheta}}{d\bar{x}} = A_{t2}\sigma'\Delta\bar{\vartheta} = 0;\ at\ z = \infty\ (\bar{t} = 0);\ \Delta V(\bar{x},0) = \Delta\bar{V}_M . \tag{11}$$

where σ′ is the heat transfer coefficient.

Solving (6) with boundary conditions (11), we finally obtain a decrease in temperature during cooling

$$\Delta\bar{\vartheta}_{cool} = \Delta\bar{\vartheta}_M \left[1 - \frac{KBi}{KBi + \frac{1}{\sqrt{\pi\bar{\tau}_{cool}}}} erf\left(\frac{\bar{x}}{2\sqrt{\bar{\tau}}} \right) \right]. \tag{12}$$

We will determine the temperature of the cooling surface under the condition $\bar{x} = 0$:

$$\Delta\bar{\vartheta}_{cool}(t) = \frac{\Delta\bar{\vartheta}_H}{1 + KBi\sqrt{\pi\bar{\tau}_{cool}}} . \tag{13}$$

Knowing the distribution of the temperature difference during heating (6) and cooling (12), one can find the thermal stresses arising in a metal friction element. For a one-dimensional problem, assuming the material is elastic and its properties do not change, the dimensionless stresses are determined using the equation

$$\bar{\sigma} = \frac{\sigma(1-\mu)}{E\alpha_C \vartheta_{aver}\bar{Q}} = \frac{\Delta\vartheta^*_{aver} - \vartheta(x,\tau)}{\vartheta_h} . \tag{14}$$

Physically, *s* is considered as the ratio of the actually acting stresses to the stresses that would occur if the thermal expansion of the metal were completely constrained [12].

The dimensionless stress at any thickness of a metal element during heating is determined by the formula

$$\bar{\sigma}^*_h = \frac{\bar{\sigma}}{Q} = \frac{1}{KBi + \frac{1}{\sqrt{\pi\bar{\tau}}}} \left[erfc\left(\frac{2}{2\sqrt{\bar{\tau}}} \right) - erfc\left(\frac{z}{2\sqrt{\bar{\tau}}} \right) + 2\sqrt{\frac{\bar{\tau}}{\pi}} \left(1 - e^{-\frac{1}{4\tau}} \right) \right] . \tag{15}$$

At *x*=0, i.e. on the working surface of the metal element

$$\bar{\sigma}^* = \frac{1}{KBi + \frac{1}{\sqrt{\pi\bar{\tau}}}} \left[2\sqrt{\frac{\bar{\tau}}{\pi}} \left(1 - e^{-\frac{1}{4\tau}} \right) - erf\left(\frac{1}{2\sqrt{\bar{\tau}}} \right) \right] \tag{16}$$

at $\bar{x} = 1{,}0$

$$\sigma^*_{2h} = \frac{2\bar{t}}{1 + KBi + \frac{1}{\sqrt{\pi\bar{\tau}}}} \left(1 - e^{\frac{1}{4\tau}} \right). \tag{17}$$

Accordingly, the temperature stress during cooling on the surface of a metal element (x=0) will be

$$\bar{\sigma}_{1cool} = \frac{KBi}{KBi + \frac{1}{\sqrt{\pi\bar{\tau}_{cool}}}}\left[\frac{2}{\sqrt{\pi}}\left(1 - e^{-\frac{1}{4\tau_{cool}}}\right) + erf\left(\frac{1}{2\sqrt{\bar{\tau}_{cool}}}\right)\right] \qquad (18)$$

at $\bar{x} = 1{,}0$

$$\bar{\sigma}^{*}_{2cool} = \frac{2}{\sqrt{\pi}}\left[\frac{KBi}{KBi + \frac{1}{\sqrt{\pi\bar{\tau}_{cool}}}}\right]\left(1 - e^{-\frac{1}{4\tau_{cool}}}\right). \qquad (19)$$

Academician A. Dzhanakhmedov established in dependencies (17 and 19) that the equivalent stresses (when heated) on the surface of the metal friction element of the braking device are almost ten times higher than the stress values arising in its 30 mm thickness.

The equivalent stress values on the heat-generating surface of a metal friction element under forced air cooling are almost five times greater than those in its total thickness. They reach a maximum during the thermal stabilization state of the metal friction element, when the gradient across its thickness becomes quasi-constant. The minimum equivalent stress values in a metal friction element occur at a steady-state surface temperature of the brake friction pairs, when the amount of heat generated is equal to the amount of heat dissipated into the environment [13].

Computational experiment. A brake pulley was chosen as the object of study.

Initial data for the working surface of the brake pulley:

A_1 = 1,14 m^2, A_2 = 1,52 m^2, $\sigma' = 60\frac{W}{(m^2 \cdot °C)}$; λ = 38 $\frac{W}{(m \cdot \deg)}$;

$a = 10{,}5 \cdot 10^{-6}\,\frac{m^2}{s}$; $\tau = 5\,s$.

We calculate

$$b_{eff} = 1{,}73\sqrt{10{,}5 \cdot 10^{-6} \cdot 5} = 12{,}5 \cdot 10^{-3}\ m, \qquad Bi = \frac{60 \cdot 12{,}5 \cdot 10^{-3}}{38} = 0{,}02$$

$$F_0 = \bar{t} = \frac{10{,}5 \cdot 10^{-6} \cdot 5}{\left(12{,}5 \cdot 10^{-3}\right)^2} = 0{,}336; \quad K = \frac{1{,}52}{1{,}14} = 1{,}33.$$

The value of the Gaussian error function erf is selected from table [14].

We determine the dimensionless temperature stress on the pulley surface during heating (16).

$$\bar{\sigma}^{*}_{1h} = \frac{1}{1{,}33 \cdot 0{,}02 + \sqrt{\frac{1}{\sqrt{3{,}14 \cdot 0{,}336}}}}\left[2\sqrt{\frac{0{,}336}{3{,}14}}\left(1 - e^{-\frac{1}{4 \cdot 0{,}336}}\right) - erf\left(\frac{1}{2\sqrt{0{,}336}}\right)\right] = 2{,}13.$$

We find the dimensionless temperature stress at full thickness during heating (17):

$$\bar{\sigma}^*{2h} = \frac{2 \cdot 0{,}336}{1 + 1{,}33 \cdot 0{,}02 + \sqrt{3.14 \cdot 0{,}336}}\left(1 - e^{-\frac{1}{4 \cdot 0{,}336}}\right) = 0{,}34$$

The thermal stress on the working surface of the pulley during forced air cooling is calculated using formula (18). Since the cooling time is t = 15 s, we first determine

$$b_{eff} = 1{,}73\sqrt{10{,}5 \cdot 10^{-6} \cdot 15} = 21{,}7 \cdot 10^{-3}\ m;$$

$$Bi = \frac{60 \cdot 21{,}7 \cdot 10^{-3}}{38} = 0{,}034;$$

$$\bar{t} = F_0 = \frac{10{,}5 \cdot 10^{-6} \cdot 15}{\left(21{,}7 \cdot 10^{-3}\right)^2} = 0{,}321.$$

Тогда

$$\bar{\sigma}^*{1res} = \frac{1{,}33 \cdot 0{,}034}{1{,}33 \cdot 0{,}034 + \frac{1}{\sqrt{3{,}14 \cdot 0{,}034}}}\left[\frac{2}{\sqrt{3{,}14}}\left(1 - e^{-\frac{1}{4 \cdot 0{,}034}}\right) + erf\left(\frac{1}{2\sqrt{0{,}034}}\right)\right] = 0{,}15\ \text{МПа}.$$

Change in temperature stresses across the full thickness of the pulley during cooling (17)

$$\bar{\sigma}^*{2cool} = \frac{2}{\sqrt{3{,}14}}\left(\frac{1{,}33 \cdot 0{,}034}{1{,}33 \cdot 0{,}34 + \frac{1}{\sqrt{3{,}14 \cdot 0{,}334}}}\right)\left(1 - e^{-\frac{1}{4 \cdot 0{,}334}}\right) = 0{,}026\text{МПа}.$$

Calculations show that the friction surface experiences the greatest thermal stresses. Cyclic thermal loading leads to fatigue stresses. The expansion of the pulley's surface layers causes compressive stresses, which are balanced by tensile stresses in the central portion of the surface. Due to shear during compression, the surface can crack. The most dangerous type of failure is crack propagation into the internal tensile region. When the pulley cools, the outer surface layers contract, causing tension. Failure in this case initiates somewhere on or near the working surface.

Based on the above, the following invention formula is proposed.

P.1. A method for the occurrence and prediction of a micro-crack network on the working surface of the brake pulley rim of a band-shoe brake of a drilling winch, comprising a rotating brake pulley and polymer linings on the brake band, the working surfaces of which are under the load of pulsed normal forces and the sliding speeds corresponding to them, characterized in that the micro-crack network on the working surface of the pulley rim is formed by pulsed normal forces, electrical and thermal currents and their discharges, temperature gradients across the width of the pulley rim, thermoelectrically stimulated polarization and depolarization processes accompanied by heating and forced cooling of the working surface of the rim, and as a consequence, the occurrence of surface (tangential) thermal stresses, and temperature gradients develop along its layers, and as a consequence, radial thermal stresses under the current aperiodic processes of heating and forced cooling of an uneven in thickness surface layer of the working surface of the pulley rim, plastic deformations of its macroprotrusions under the action of pulsed specific loads causing displacement of the upper layer relative to the underlying one, which are subject to a thermal stress gradient with subsequent imposition of residual thermal stresses on them, and temperature gradients across the thickness of the brake pulley rim layers contribute to the expansion of microcracks along their length and depth.

P.2. The method for the occurrence and prediction of a network of micro-cracks on the working surface of the rim of a metal friction element according to p.1, characterized in that by the difference in the values of tangential ($d\sigma_r/dl$) and radial ($d\sigma_r/d\delta$) thermal stress gradients developed on the friction surface of the rim along its thickness, and at the same time they are interconnected and are in dependencies that predict the occurrence and growth of a network of micro-cracks: stable - ($d\sigma_r/dl$) - ($d\sigma_r/d\delta$) $\geq$ ($d\sigma_r/dl$); slow ($d\sigma_r/d\delta$) $\rightarrow$ to ($d\sigma_r/dl$) and accelerated - ($d\sigma_r/d\delta$) » ($d\sigma_r/dl$).

Features of the finite element modeling method using the Ansys Workbench program for the stress state of metal friction elements. The finite element method (FEM) is an effective numerical method for solving engineering and physics problems. Many methods are based on replacing a continuous model with a discrete model, which has a finite number of unknowns. Because the number of unknowns can be very large, requiring repeated computational procedures, numerical methods are most often applied using computers. Computers are used to analyze the stress state of metal friction elements and to estimate the high-speed air currents that wash over their surfaces. Modern computer programs that implement FEM include Ansys®, Nastrsn®, Cosmos®, CosmosWorks®, Cosmos FlowWorks®, and ICEM CFD®. In structural modeling, MSE is very helpful in assessing the force and stiffness visualizations at the locations of displacements and bends and the distribution of forces and displacements of microprotrusions of a metal friction element at the micro- and nano-levels.

The basic idea of the finite element method is that any continuous quantity (displacement, force, specific loads, surface temperatures, etc.) can be approximated by a model composed of individual elements (sections). At each of these elements, the continuous quantity being studied is approximated by a partially continuous function, which is constructed from the values of the continuous quantity being studied.

The ANSYS system is designed to solve problems in the mechanics of deformed solids (DSS) and their surface temperatures under high-speed air currents. Furthermore, the system allows for solving problems for various states of a solid subjected to mechanical and thermal loading, taking into account the consequences of their action on its surface [15].

Due to the development and widespread use of 3D modeling tools, importing previously created files appears to be the most convenient option for creating computational models that can be used in scientific research practice.

Ansys Workbench is a new generation of software products based on a modern object-oriented approach to deep analysis, leveraging the capabilities of Ansys. This area of deep analysis offers unique integration capabilities with CAD systems (including bidirectional associative linking). It allows you to combine the design process in a CAP package with the generation of reliable calculation data and design optimization.

Workbench Products consists of the following modules: Design Simulation (uses DesignSpace, Professional, Structural, Mechanical licenses), Design Modeler, Design Xplorer, FE Modeler.

To calculate the stress-strain state of self-ventilated brake discs, a method for solving scientific and practical problems was used. In the calculation and design of self-ventilated brake discs, a combination of computer three-dimensional parametric modeling and the finite element method is particularly effective. The former allows for varying the design parameters of the brake disc (thickness and outer diameter of the disc) without the need to rebuild the model as a whole and establishing the influence of these parameters on the stress-strain state of the brake disc using the finite element method. The latter is based on the criterion of the specific potential energy of deformation of the brake disc in accordance with the fourth theory of strength [9]. According to this theory, a dangerous state (yield) generally occurs when the specific potential energy of deformation reaches its limit value. This state is determined by simple stretching of the disc element located on the friction belt, at the moment of its yielding, that is, under the condition:

$$U_e = U_{e.m} \tag{20}$$

when the strength condition is written as:

$$U_e = [U_e] \tag{21}$$

$$U_\partial = \frac{1+\mu}{3E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma 2 + \sigma_2\sigma 3 + \sigma_3\sigma 1)\right] \tag{22}$$

where μ is Poisson's ratio; E is the modulus of elasticity; $\sigma_1, \sigma_2, \sigma_3$ are the components of the normal stress of the elementary volume of the disk.

For simple tension at the yield point ($\sigma_1 = \sigma_m, \sigma_2 = \sigma 3 = 0$) we have

$$U_{a.\partial} = \frac{1+\mu}{3E}\sigma_\partial^2 \tag{23}$$

Thus, condition (21) after substituting expressions (22) and (23) turns into the following expression

$$\sqrt{\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma 2 + \sigma_2\sigma 3 + \sigma_3\sigma 1)\right]} = \sigma_m \tag{24}$$

or

$$\sqrt{0{,}5\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]} = \tau_m \tag{25}$$

The strength condition will be as follows

$$\sqrt{0{,}5\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]} \le \sigma_m \tag{26}$$

Therefore, the equivalent stresses according to the fourth theory of strength are equal to

$$\sigma_{equIV} = \sqrt{0{,}5\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]} \tag{27}$$

It should be noted that the expression σ_{equIV} coincides with the expression for determining the stress intensity σ_U.

Let's consider the basic considerations for calculating the elastic limits of round rotating discs, taking into account axisymmetric heating due to frictional interaction between their friction zones and the working surfaces of the brake pads. The static coefficient of mutual overlap does not exceed 0.2.

It is known that if a disk of constant thickness has an outer diameter that is four or more times greater than α, then, according to the exact solution of elasticity theory, it can be assumed that mechanical stresses are uniformly distributed across the thickness of the disk's circumferential and radial sections, and that the individual circular layers of the disk, while deforming in the same manner, are not in force interaction with one another (i.e., a plane stress state exists). Two hypotheses are used in this regard. The first hypothesis states that when calculating thin disks with an arbitrary profile in the radial section, a uniform stress distribution law across the disk's thickness is assumed. According to the second hypothesis, a plane stress state exists, i.e., stress does not arise in areas parallel to the disk's midplane.

It is assumed that under steady-state thermal conditions the temperature across the thickness of the disk is constant and equal to the temperature at the corresponding point on the surface.

Brake pads with friction linings are pressed by their working surfaces, which have a segmented frontal appearance, under the influence of normal forces from both directions. At the beginning and end of braking, these forces act with minimum and maximum loads. At all times, the working surfaces of the linings interact frictionally with the fresh friction track of the brake disc. These processes continue for one revolution of the disc, and they are repeated on the second revolution. Moreover, above the upper circumference of the friction track, there is significantly less matte surface area on the side surfaces of the disc than below the lower circumference, resulting in significantly different metal contents of the brake disc components.

Discussion of results. Theoretical research and a computational experiment applied to local equivalent stresses in the metal friction element of braking devices allowed us to propose:

- a flowchart for assessing the energy load of friction pairs, in which the equivalent stress assessment is presented last;
- a mathematical description of the relationship between local temperature zones and equivalent stresses in friction pairs;
- a computational and experimental approach to determining equivalent stresses on the working surface of metal friction elements;
- a patent claim for "Method for the occurrence and prediction of a microcrack network on the working surface of metal elements at different rates." Method: finite element modeling of the stress state of metal friction elements using Ansys Workbench.

Conclusions. Thus, methods for determining equivalent stresses in friction pairs of braking devices are proposed.

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ЛОКАЛЬНІ ЕКВІВАЛЕНТНІ НАПРУЖЕННЯ У МЕТАЛЕВИХ ФРИКЦІЙНИХ ЕЛЕМЕНТАХ ГАЛЬМОВИХ ПРИСТРОЇВ

Теоретичні дослідження і обчислювальний експеримент стосовно локальних еквівалентних напружень металевих фрикційних елементів гальмових пристроїв дозволили встановити і запропонувати блок-схему оцінки енергонавантаженості пар тертя, в якій на останньому місці представлено оцінку еквівалентних напружень. Математичним описом є взаємозв'язок між локальними-зонами температури і еквівалентними напруженнями в парах тертя. Обчислювальний та експеримент з визначення еквівалентних напружень на робочій поверхні металевих фрикційних елементів. Формулу на патент «Спосіб виникнення та прогнозування сітки мікротріщин на робочій поверхні металевих елементів з різним темпом». Метод: звичайно-елементного моделювання за допомогою програми «Ansys Workbench» напруженого стану металевих фрикційних елементів.

Ключові слова: гальмовий пристрій, пара тертя, металевий елемент, теплообмінні процеси, напруження.

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